Blok 2 - Vaardigheden

bladzijde 122

1a $9-x^2 \geq 0$ geeft $x^2 \leq 9$ dus $-3 \leq x \leq 3$. Dus $D_f = [-3, 3]$
b $x^2-1 \geq 0$ geeft $x^2 \geq 1$ dus $x \leq -1$ of $x \geq 1$. Dus $D_f = \langle \leftarrow, -1] \cup [1, \rightarrow\rangle$
c $x^2+5>0$ geeft $x^2 > -5$. Dus x kan elke waarde aannemen. $D_f = \mathbb{R}$
d $x^2-4x-5 \geq 0$ geeft $(x-5)(x+1) \geq 0$ dus $x \leq -1$ of $x \geq 5$. $D_f = \langle \leftarrow, -1] \cup [5, \rightarrow\rangle$
e $6-x^2>0$ geeft $x^2<6$ dus $-\sqrt{6} < x < \sqrt{6}$. $D_f = \langle -\sqrt{6}, \sqrt{6}\rangle$
f $x^4-2x^3 \geq 0$ geeft $x^3(x-2) \geq 0$ dus $x \leq 0$ of $x \geq 2$. $D_f = \langle \leftarrow, 0] \cup [2, \rightarrow\rangle$

2a $x^2+2x+10 \geq 0$ voor elke waarde van x. $D_f = \mathbb{R}$ en $B_f = [3, \rightarrow\rangle$
b $D_f = \mathbb{R}$ en $B_f = \langle 0, \frac{1}{3}]$
c $1-\frac{2}{x} \geq 0$ geeft $\frac{2}{x} \leq 1$ dus $x<0$ of $x \geq 2$. $D_f = \langle \leftarrow, 0\rangle \cup [2, \rightarrow\rangle$ en $B_f = [0, 1\rangle \cup \langle 1, \rightarrow\rangle$
d $x \geq 0$ en $2-x>0$ dus $x \geq 0$ en $x<2$. $D_f = [0, 2\rangle$ en $B_f = [0, \rightarrow\rangle$

3a $D_h = [1, \rightarrow\rangle$
b De uitkomst van $2\sqrt{x-1}$ is altijd groter of gelijk aan nul dus het bereik van h is $[1, \rightarrow\rangle$.
c Het randpunt is $(1, 1)$.

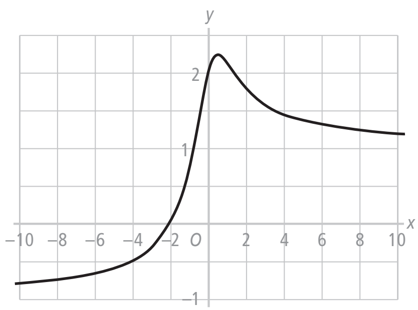
bladzijde 123

4a $\sqrt{x+2} = x \Rightarrow x+2 = x^2 \Rightarrow x^2-x-2=0 \Rightarrow (x+1)(x-2)=0 \Rightarrow x=-1$ of $x=2$
$x=-1$ voldoet niet dus de oplossing is $x=2$
b $\sqrt{x+2} = x-4 \Rightarrow$
$x+2 = x^2-8x+16 \Rightarrow x^2-9x+14=0 \Rightarrow (x-2)(x-7)=0 \Rightarrow x=2$ of $x=7$
$x=2$ voldoet niet dus de oplossing is $x=7$
c $3\sqrt{x}-x=2 \Rightarrow 3\sqrt{x}=x+2 \Rightarrow 9x=x^2+4x+4 \Rightarrow x^2-5x+4=0 \Rightarrow$
$(x-1)(x-4)=0 \Rightarrow x=1$ of $x=4$
d $\frac{3\sqrt{x}}{2} = x \Rightarrow 3\sqrt{x}=2x \Rightarrow 9x=4x^2 \Rightarrow 4x^2-9x=0 \Rightarrow x(4x-9)=0 \Rightarrow x=0$ of $x=2\frac{1}{4}$
e $3\sqrt{x+2}-2=-x \Rightarrow 3\sqrt{x+2}=-x+2 \Rightarrow 9(x+2)=x^2-4x+4 \Rightarrow x^2-13x-14=0 \Rightarrow$
$(x-14)(x+1)=0 \Rightarrow$
$x=14$ of $x=-1$
$x=14$ voldoet niet dus de oplossing is $x=-1$
f $\frac{5}{2\sqrt{x}}+\frac{2}{\sqrt{x}}=\frac{9}{4} \Rightarrow \frac{5}{2\sqrt{x}}+\frac{4}{2\sqrt{x}}=\frac{9}{2\sqrt{x}}=\frac{9}{4} \Rightarrow 2\sqrt{x}=4 \Rightarrow \sqrt{x}=2 \Rightarrow x=4$

5a Nee, $f(x)=g(x)$ alleen voor $x=0$.
b Nee, $f(x)=g(x)$ alleen voor $x>0$.
c Ja, $f(x)=g(x)$ want $\sqrt{4x^3}=\sqrt{4}\cdot\sqrt{x^3}=2(x^3)^{\frac{1}{2}}=2x^{1\frac{1}{2}}=2x\sqrt{x}$

6a domein $=\mathbb{R}$ want $x^2+1>0$ voor elke waarde van x
b $x+2=0$ als $x=-2$

c



$y = 1$ en $y = -1$ zijn de horizontale asymptoten

d $\dfrac{x+2}{\sqrt{x^2+1}} = \sqrt{5} \Rightarrow \sqrt{5} \cdot \sqrt{x^2+1} = x+2 \Rightarrow 5\left(x^2+1\right) = x^2+4x+4 \Rightarrow$

$4x^2 - 4x + 1 = 0 \Rightarrow x^2 - x + \frac{1}{4} = 0 \Rightarrow$

$\left(x - \frac{1}{2}\right)^2 = 0 \Rightarrow x = \frac{1}{2}$

Het maximum van de functie ligt bij $x = \frac{1}{2}$ dus $f(x) \geq \sqrt{5}$ voor $x = \frac{1}{2}$.

e De lijn $y = \sqrt{5}$ gaat door de top van de grafiek.

bladzijde 124

7a $\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2} = \frac{3}{2}\sqrt{2} = 1\frac{1}{2}\sqrt{2}$

b $\frac{6}{\sqrt{3}} = \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = \frac{6}{3}\sqrt{3} = 2\sqrt{3}$

c $\left(\frac{2}{3}\sqrt{3}\right)^4 = \left(\frac{2}{3}\right)^4 \cdot \left(\sqrt{3}\right)^4 = \frac{16}{81} \cdot 9 = \frac{16}{9} = 1\frac{7}{9}$

d $\sqrt{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2} + \frac{\sqrt{2}}{2} = \sqrt{2} + \frac{1}{2}\sqrt{2} = 1\frac{1}{2}\sqrt{2}$

e $\sqrt{1875} = \sqrt{625} \cdot \sqrt{3} = 25\sqrt{3}$

f $\sqrt{2048} = \sqrt{1024} \cdot \sqrt{2} = 32\sqrt{2}$

8a $\dfrac{\frac{6}{\sqrt{x}}}{2x} = \dfrac{6}{\sqrt{x}} \cdot \dfrac{1}{2x} = \dfrac{3}{x\sqrt{x}}$

b $\frac{1}{2}x \cdot \dfrac{2}{\sqrt{x}} = \dfrac{x}{\sqrt{x}} = \sqrt{x}$

c $\dfrac{x-3}{\sqrt{x}} + \dfrac{2}{x\sqrt{x}} = \dfrac{x(x-3)}{x\sqrt{x}} + \dfrac{2}{x\sqrt{x}} = \dfrac{x^2-3x+2}{x\sqrt{x}}$

d $\dfrac{x-1}{\sqrt{x}} \cdot \dfrac{2\sqrt{x}}{x+1} = \dfrac{2x-2}{x+1}$

9a $x^2 - 4x + 2 = 0$

$x = \dfrac{4-\sqrt{8}}{2}$ of $x = \dfrac{4+\sqrt{8}}{2}$

b $\dfrac{4 \pm \sqrt{8}}{2} = \dfrac{4 \pm \sqrt{4} \cdot \sqrt{2}}{2} = \dfrac{4 \pm 2\sqrt{2}}{2} = \frac{4}{2} \pm \dfrac{2\sqrt{2}}{2} = 2 \pm \sqrt{2}$

c $\dfrac{6+\sqrt{45}}{3} = \dfrac{6+\sqrt{9} \cdot \sqrt{5}}{3} = \dfrac{6}{3} + \dfrac{3\sqrt{5}}{3} = 2 + \sqrt{5}$

d $\dfrac{8-\sqrt{112}}{12} = \dfrac{8-\sqrt{16} \cdot \sqrt{7}}{12} = \dfrac{8}{12} - \dfrac{4}{12}\sqrt{7} = \frac{2}{3} - \frac{1}{3}\sqrt{7}$

10a $\sqrt{x^2-4} = x-2$

$x^2 - 4 = x^2 - 4x + 4$

$4x = 8$

$x = 2$

b $2\sqrt{x} = \sqrt{2x}$

$4x = 2x$

$x = 0$

c $x\sqrt{x} = 3x$
$x^2 \cdot x = 9x^2$
$x^3 - 9x^2 = 0$
$x^2(x-9) = 0$
$x = 0$ of $x = 9$

d $\frac{2}{\sqrt{3}} = \frac{2\sqrt{x}}{x^2}$
$2\sqrt{x} \cdot \sqrt{x^3} = 2x^2$
$2\sqrt{x^4} = 2x^2$
$4x^4 = 4x^4$ mits $x > 0$
dus $x > 0$

e $\sqrt{\frac{x}{4}} \cdot \sqrt{4x} = x$
$\sqrt{\frac{x}{4} \cdot 4x} = x$
$\sqrt{x^2} = x$
$x = x$ mits $x \geq 0$
dus $x \geq 0$

f $\sqrt{\frac{4}{x}} = \frac{2}{\sqrt{x}}$
$\frac{4}{x} = \frac{4}{x}$ mits $x > 0$
dus $x > 0$

bladzijde 125

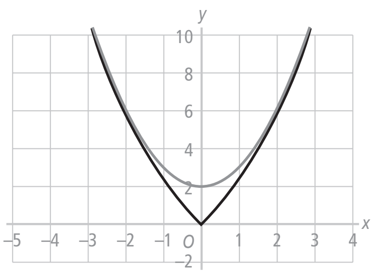
11a $f(x) = 4x^{-1\frac{1}{2}}$ dus $f'(x) = -6x^{-2\frac{1}{2}} = -\frac{6}{x^2\sqrt{x}}$
$f'(4) = -\frac{6}{16\sqrt{4}} = -\frac{3}{16}$

b $\frac{4}{x\sqrt{x}} = \sqrt{x} \Rightarrow x\sqrt{x} \cdot \sqrt{x} = 4 \Rightarrow x^2 = 4 \Rightarrow x = 2$ of $x = -2$
$x = -2$ voldoet niet dus $x = 2$
$g(2) = \sqrt{2}$ dus $S(2, \sqrt{2})$

c $g'(x) = \frac{1}{2\sqrt{x}}$ dus $g'(2) = \frac{1}{2\sqrt{2}}$
$y = \frac{1}{2\sqrt{2}}x + b$ $(2, \sqrt{2})$
$\sqrt{2} = \frac{1}{2\sqrt{2}} \cdot 2 + b$ dus $b = \sqrt{2} - \frac{1}{\sqrt{2}} = \sqrt{2} - \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2}$
$y = \frac{1}{2\sqrt{2}}x + \frac{1}{2}\sqrt{2}$

d $f'(4) = -\frac{6}{32}$
$y = -\frac{6}{32}x + b$ $(4, \frac{1}{2})$
$\frac{1}{2} = -\frac{6}{32} \times 4 + b$ dus $b = 1\frac{1}{4}$
Dus $y = -\frac{6}{32}x + 1\frac{1}{4}$ is inderdaad de lijn die de grafiek van *f* raakt in *A*.

12a



b $\sqrt{x^4 + 4x^2} = x^2 + 4 \Rightarrow x^4 + 4x^2 = x^4 + 8x^2 + 16 \Rightarrow -4x^2 = 16 \Rightarrow x^2 = -4$ dus geen oplossingen
De grafieken van *f* en *g* hebben geen snijpunten.

c $x^2 + 2 - \sqrt{x^4 + 4x^2} = \frac{1}{2} \Rightarrow x^2 + 1\frac{1}{2} = x^4 + 4x^2 \Rightarrow x^4 + 3x^2 + 2\frac{1}{4} = x^4 + 4x^2 \Rightarrow 2\frac{1}{4} = x^2 \Rightarrow$
$x = 1\frac{1}{2}$ of $x = -1\frac{1}{2}$

d Uit de grafiek volgt $\langle -1\frac{1}{2}, 1\frac{1}{2} \rangle$.

13a $\frac{3}{\sqrt{2x+4}}=2\Rightarrow \sqrt{2x+4}=1\frac{1}{2}\Rightarrow 2x+4=2\frac{1}{4}\Rightarrow 2x=-1\frac{3}{4}\Rightarrow x=-\frac{7}{8}$

b $\sqrt{x^2+2x}=x+2\Rightarrow x^2+2x=x^2+4x+4\Rightarrow -2x=4\Rightarrow x=-2$

c $\frac{2+x}{\sqrt{x}}=3\Rightarrow 3\sqrt{x}=2+x\Rightarrow 9x=4+4x+x^2\Rightarrow x^2-5x+4=0\Rightarrow (x-1)(x-4)=0\Rightarrow$
$x=1$ of $x=4$

d $2+\sqrt{x}=x\Rightarrow \sqrt{x}=x-2\Rightarrow x=x^2-4x+4\Rightarrow x^2-5x+4=0\Rightarrow (x-1)(x-4)=0\Rightarrow$
$x=1$ of $x=4$
$x=1$ voldoet niet dus $x=4$ is de oplossing

e $\sqrt{6+\sqrt{x}}=4\Rightarrow 6+\sqrt{x}=16\Rightarrow \sqrt{x}=10\Rightarrow x=100$

f $x\sqrt{x}=\sqrt{11}\Rightarrow x^{1\frac{1}{2}}=\sqrt{11}\Rightarrow x=\left(11^{\frac{1}{2}}\right)^{\frac{1}{1\frac{1}{2}}}=11^{\frac{1}{3}}=\sqrt[3]{11}$

14a $g'(x)=-2x+\frac{1}{\sqrt{x}}$ dus $g'(4)=-8+\frac{1}{2}=-7\frac{1}{2}$
$y=-7\frac{1}{2}x+b$ $(4,-12)$
$-12=-30+b$ geeft $b=18$ dus $y=-7\frac{1}{2}x+18$

b $h'(x)=\frac{1}{3\sqrt{x}}$ dus $h'(9)=\frac{1}{9}$
$y=\frac{1}{9}x+b$ $(9,1)$
$1=1+b$ geeft $b=0$ dus $y=\frac{1}{9}x$

c $f'(x)=\frac{3}{2\sqrt{x}}$ dus $f'(1)=1\frac{1}{2}$
$y=1\frac{1}{2}x+b$ $(1,5)$
$5=1\frac{1}{2}+b$ geeft $b=3\frac{1}{2}$ dus $y=1\frac{1}{2}x+3\frac{1}{2}$

d $r'(x)=\frac{1}{2x\sqrt{x}}-\frac{2}{x^2}+\frac{6}{x^3}$ dus $r'(1)=4\frac{1}{2}$
$y=4\frac{1}{2}x+b$ $(1,-2)$
$-2=4\frac{1}{2}+b$ geeft $b=-6\frac{1}{2}$ dus $y=4\frac{1}{2}x-6\frac{1}{2}$

15a $-2x+3\sqrt{x}=0$
$3\sqrt{x}=2x$
$9x=4x^2$
$4x^2-9x=0$
$x(4x-9)=0$
$x=0$ of $x=2\frac{1}{4}$
$(0,0)$ en $(2\frac{1}{4},0)$

b Eerst de vergelijking oplossen.
$-2x+3\sqrt{x}=-2$
$3\sqrt{x}=-2+2x$
$9x=4-8x+4x^2$
$4x^2-17x+4=0$
$x^2-4\frac{1}{4}x+1=0$
$(x-4)(x-\frac{1}{4})=0$
$x=4$ of $x=\frac{1}{4}$ ($x=\frac{1}{4}$ voldoet niet)
Uit de plot volgt $[0,4\rangle$.

c $f'(x)=-2+\frac{3}{2\sqrt{x}}$

d $-2+\frac{3}{2\sqrt{x}}=0$
$\frac{3}{2\sqrt{x}}=2$
$4\sqrt{x}=3$
$16x=9$
$x=\frac{9}{16}$
$f(\frac{9}{16})=1\frac{1}{8}$
In punt $(\frac{9}{16},1\frac{1}{8})$ is de helling nul