Game Theory

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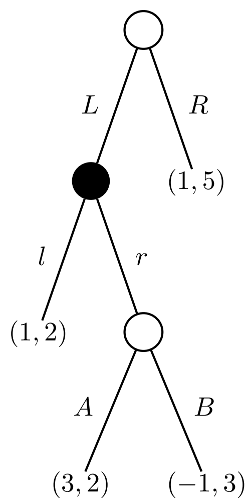
Exercise set 3

Exercise 1 *Consider the following* 2*-player extensive form game given by the game tree*



a. *How many, and which, strategies does player* 1 *have? How many, and which, strategies does player* 2 *have?*

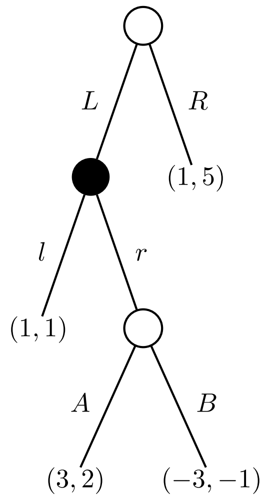
b. *Give a completely elaborated plan of playing for player* 1 *that is not a strategy.*

c. *Determine a normal form for this game.*

d. *Determine for each player the strictly dominant strategies.*

e. *Determine the Nash equilibria.*

Exercise 2 *Consider the following* 2*-player extensive form game given by the game tree*



a. Show that there are three Nash equilibria.

b. Which Nash equilibrium is "the best"?

Exercise 3 *Consider the following game between two (rational and intelligent) players. There is a pillow with* 100 *matches. They alternately remove* 1, 2 *or* 4 *matches from it. (Player* 1 *begins.) The player who makes the last move wins. What is the value of this game?*

Short solutions.

Solution 1 a. Player 1 has 4 strategies and player 2 has 2 strategies.

b. Playing R.

c. This is the bimatrix game $\begin{pmatrix} & l & r \\ LA & 1;2 & 3;2 \\ LB & 1;2 & -1;3 \\ RA & 1;5 & 1;5 \\ RB & 1;5 & 1;5 \end{pmatrix}$.

d. There are no strictly dominant strategies.

e. $(LA,l),(RA,l),(RB,l)$ and (LA,r).

Solution 2 a. A normal form is $\begin{pmatrix} & l & r \\ LA & 1;1 & 3;2 \\ LB & 1;1 & -3;3 \\ RA & 1;5 & 1;5 \\ RB & 1;5 & 1;5 \end{pmatrix}$. Nash equilibria: $(RA,l),(RB,l)$ and (LA,r).

b. (LA,r). Reason: if player 1 has to move for the second time, then he plays A. Player 2 is aware of this, and therefore, if he has to move, plays r. Player 1 is aware of this and therefore plays L as first move.

Solution 3 The loosing positions are those with number of matches that when divided by 3 has remainder 0. As 100 divided by 3 has remainder 1, player 1 can win. So the value is $+1$.