Game Theory

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Exercise set 1

Exercise 1 *This exercise deals with the Tic-Tac-Toe game. Lesson 1 contains a link where You can play the game. For the first two parts of this exercise You need an opponent.*

a. *Play the game several times with an opponent. Be sure that (after some plays, may be also at home) You can play this game in such a way that You do not loose.*

b. *Both players write on a paper a completely elaborated plan of play, i.e. write down how they will play the game. Then they put both papers on the table and play the game using these strategies. Note: the plan of play should be completely clear: say a third person (called 'referee') can play the game for You, without asking You what You mean or taking a decision how to play.*

c. *Now suppose You are player* 1 *(i.e. You begin). Try to find a completely elaborated plan of play that guarantees You at least a draw.*

Exercise 2 *This exercise deals with the Hex game. For the first part of this exercise You need an opponent. (Lesson 1 contains a link where You can play the game against a computer program.)*

a. *Play the game various times with an opponent.*

b. *As explained in Lesson 1, it impossible (and quite surprising) that this game cannot end in a draw (i.e. no winner, no loser). Please convince Yourself that this statement is true by trying to obtain a draw.*

Exercise 3 *Consider Tic-Tac-Toe (with the standard numbering of cells). Consider the following completely elaborated plan of play player* 1.

First move in cell 5. *Each following move according to the first description in the following list that can be applied:*

(1) Lowest number in same row in which opponent did last move.
(2) Lowest number in same column in which opponent did last move.
(3) Lowest number.

Give a completely elaborated plan of play of player 2 *that wins from that of player* 1.

Exercise 4 *Show that the claim made in the common knowledge illustration in Lesson 1 is true if $M = 1$ and if $M = 2$.*

Exercise 5 *Given the following bi-matrix-game:*

$$\begin{pmatrix} 3;8 & 4;8 & 2;3 \\ 1;7 & 2;6 & 8;1 \\ 3;4 & 4;4 & 2;2 \\ 1;1 & 1;-1 & 1;-1 \end{pmatrix}.$$

a. Determine the strictly dominant strategies.

b. Determine the Nash equilibria.

c. Determine the weakly Pareto efficient strategy profiles.

d. Determine the fully cooperative strategy profiles.

Short solutions.

Solution 1 c. Number the cells from left to right and from top to bottom with $1, 2, \ldots, 9$ First move: 5. For each next move, if it applies, move opposite to last own move (and then win), otherwise move clockwise beside last move of opponent and if this is not possible, then move anti-clockwise beside last move of opponent.

Solution 2 I hope that You enjoyed trying.

Solution 3 Consider the following completely elaborated plan of play: $(3, 5, 1, 9, 7, 6, 2, 4, 8)$, i.e. player moves in first free cell number that occurs in this sequence. Player 2 wins.

Solution 4 Case where $M = 1$: there is 1 dwarf with a red cap. As this dwarf does not see another dwarf with a red cap and knows that there is at least one with a red cap, he can conclude that he has a red cap. So he stands up when the bell rings for the first time. Note that this conclusion does not hold if there are two dwarfs with a red cap. The other dwarfs (having all green caps) do not stand up.

Case where $M = 2$: there are 2 dwarfs with a red cap. So each of them sees 1 dwarf with a red cap. When the bell rings for the first time, no dwarf stands up. Because of this, the 2 dwarfs can conclude that they have a red cap and will stand up when the bell rings for the second time.

Solution 5 a. No player has a strictly dominant strategy.

b. Nash equilibria: $(1, 1)$ (i.e. row 1 and column 1), $(1, 2), (3, 1), (3, 2)$.

c. $(1, 1), (1, 2), (2, 3), (3, 2)$.

d. $(1, 2)$.