Advanced Microeconomics

UEC-51806

Part 1: Consumer Theory and Theory of the Firm

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The material contained in these slides draws heavily on:
Geoffrey A. Jehle and Philip J. Reny (2011). Advanced Microeconomic Theory (3rd Edition). Prentice Hall, 672 p.

Disclaimer

These slides are not meant to be your sole study material.

They are just a (incomplete) summary of what will be covered in Part 1 of the course.

Quotes to live by

“You've got to be very careful if you don't know where you are going, because you might not get there.”

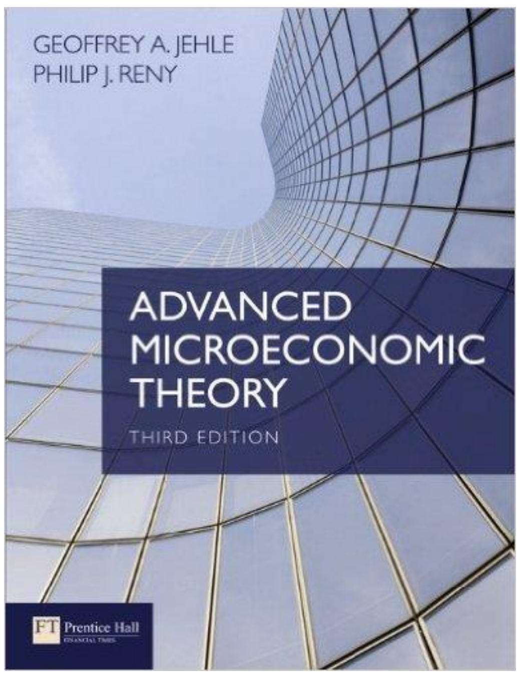
“I'm not going to buy my kids an encyclopedia. Let them walk to school like I did.”

- Yogi Berra

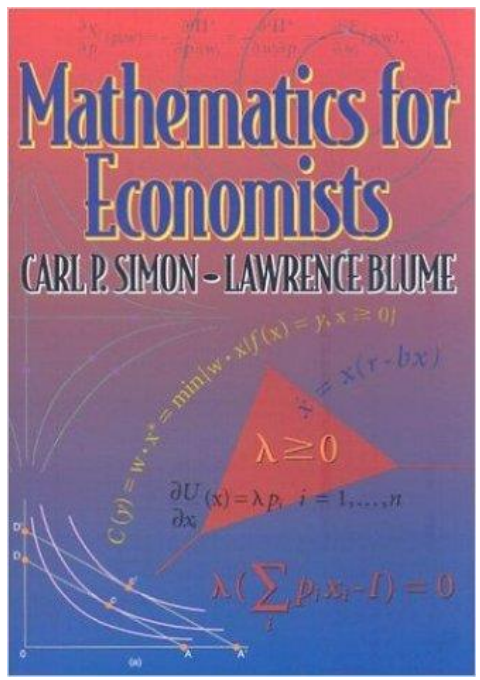
General Information

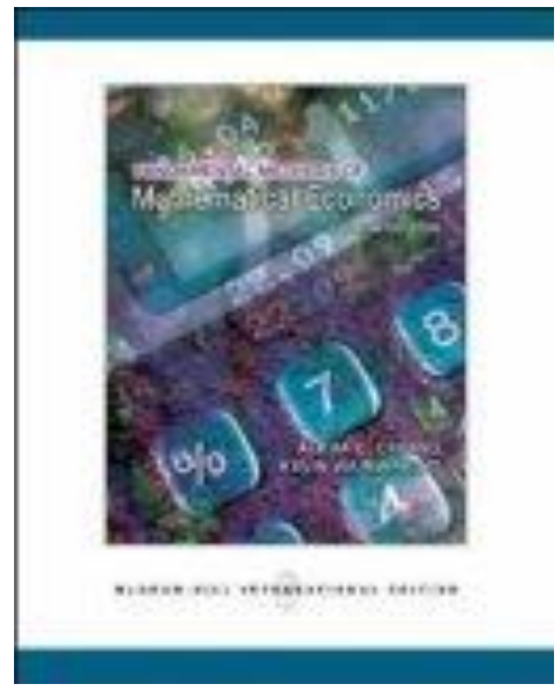
- Advance level course (abstraction, mathematically oriented, but fun)
- All relevant information (e.g., problem sets, home assignments, tests) are on Brightspace

Textbook



Recommended supplemental reading





General Information

- Exercises to be solved at home and discussed in class (not graded)
- Home assignment for Part 1 due on or before October 2, 2023. Hand-written only.
- No official office hours for Part 1. Please send me an e-mail to schedule an online meeting

Consumer Theory

Primitive notions

Four building blocks:

- Consumption (choice) set
- Consumption bundle (plan)
- Preference relation
- Behavioral assumption

Properties of the Consumption Set, X

The minimal requirements on the consumption set are

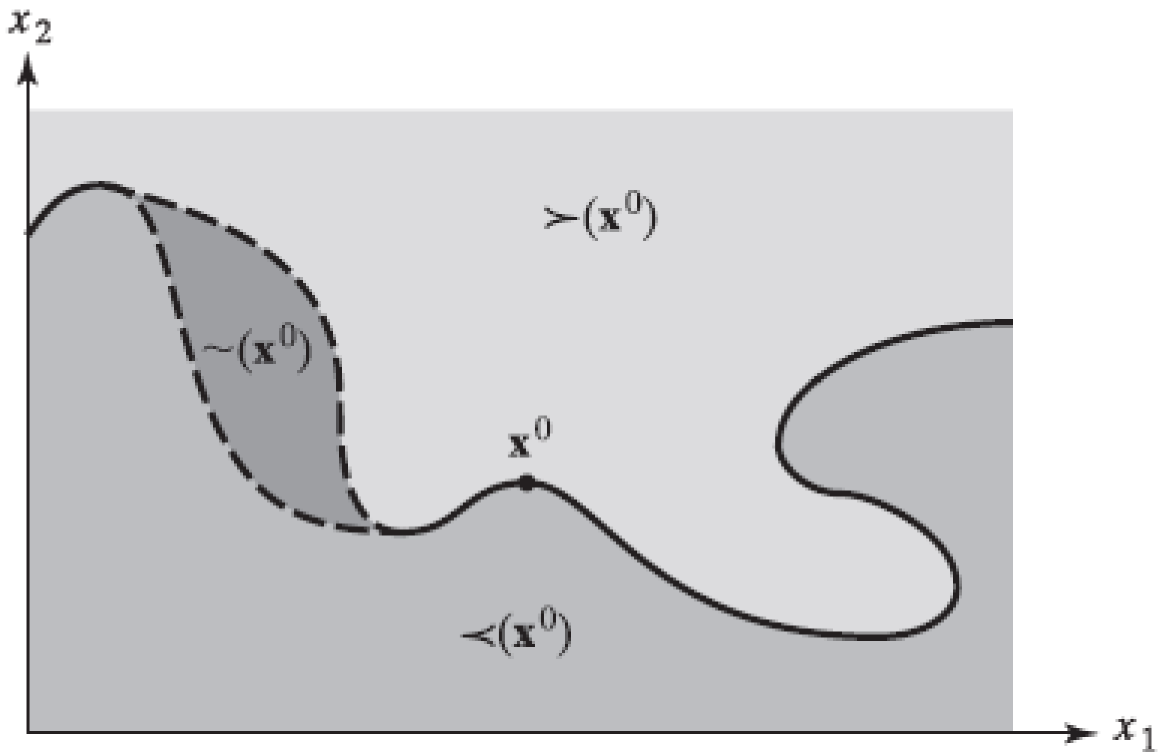
1. $X \subseteq \mathbb{R}^n_+$.
2. X *is closed.*
3. X *is convex.*
4. $\mathbf{0} \in X$.

Preferences and utility

Axioms of consumer choice

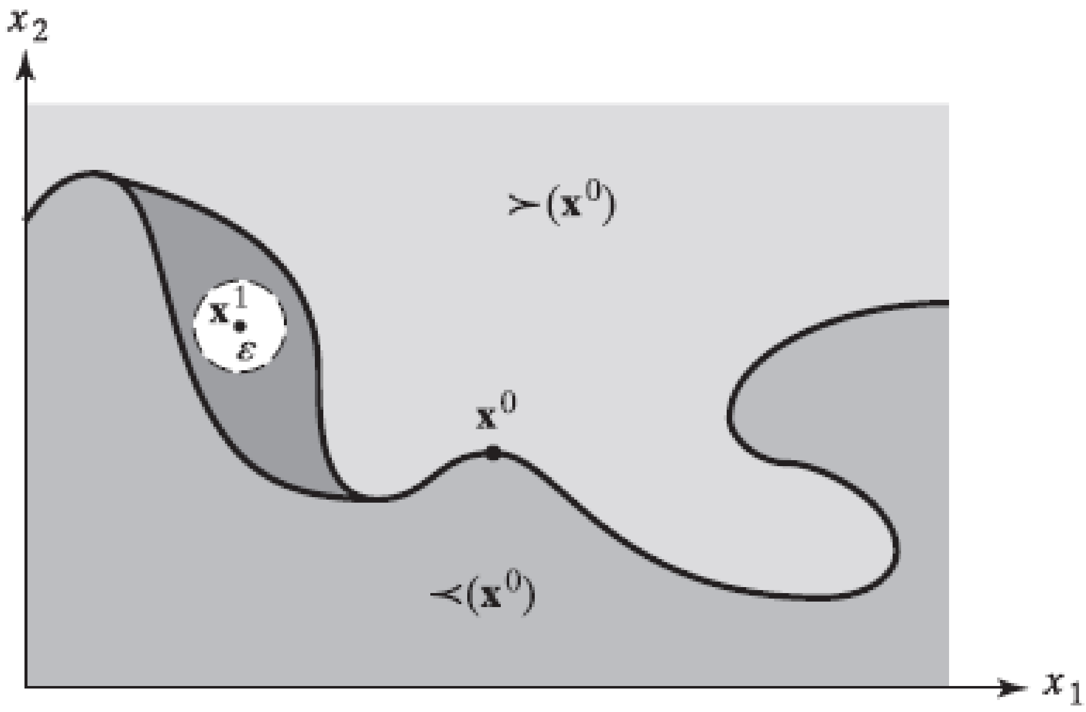
AXIOM 1: Completeness. *For all* $\mathbf{x}^1$ *and* $\mathbf{x}^2$ *in X, either* $\mathbf{x}^1 \succsim \mathbf{x}^2$ *or* $\mathbf{x}^2 \succsim \mathbf{x}^1$.

AXIOM 2: Transitivity. *For any three elements* $\mathbf{x}^1$, $\mathbf{x}^2$, *and* $\mathbf{x}^3$ *in X, if* $\mathbf{x}^1 \succsim \mathbf{x}^2$ *and* $\mathbf{x}^2 \succsim \mathbf{x}^3$, *then* $\mathbf{x}^1 \succsim \mathbf{x}^3$.



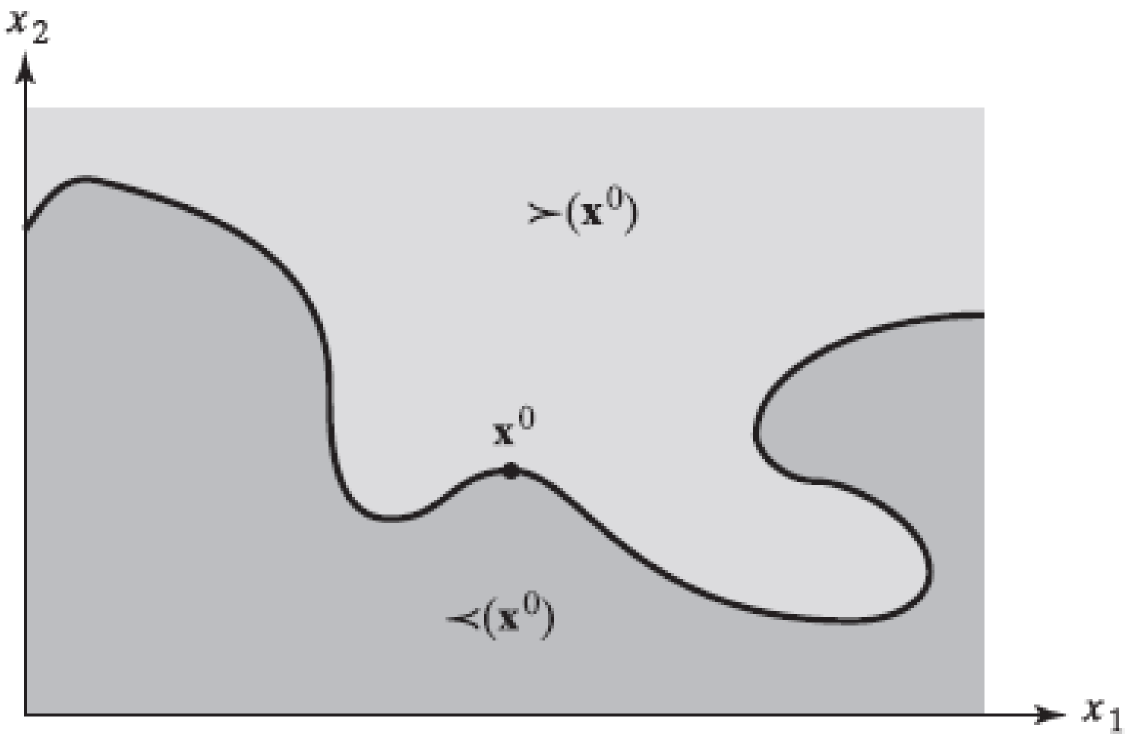
Preferences and utility

AXIOM 3: Continuity. *For all* $\mathbf{x} \in \mathbb{R}^n_+$, *the 'at least as good as' set,* $\succsim(\mathbf{x})$, *and the 'no better than' set,* $\precsim(\mathbf{x})$, *are closed in* $\mathbb{R}^n_+$.



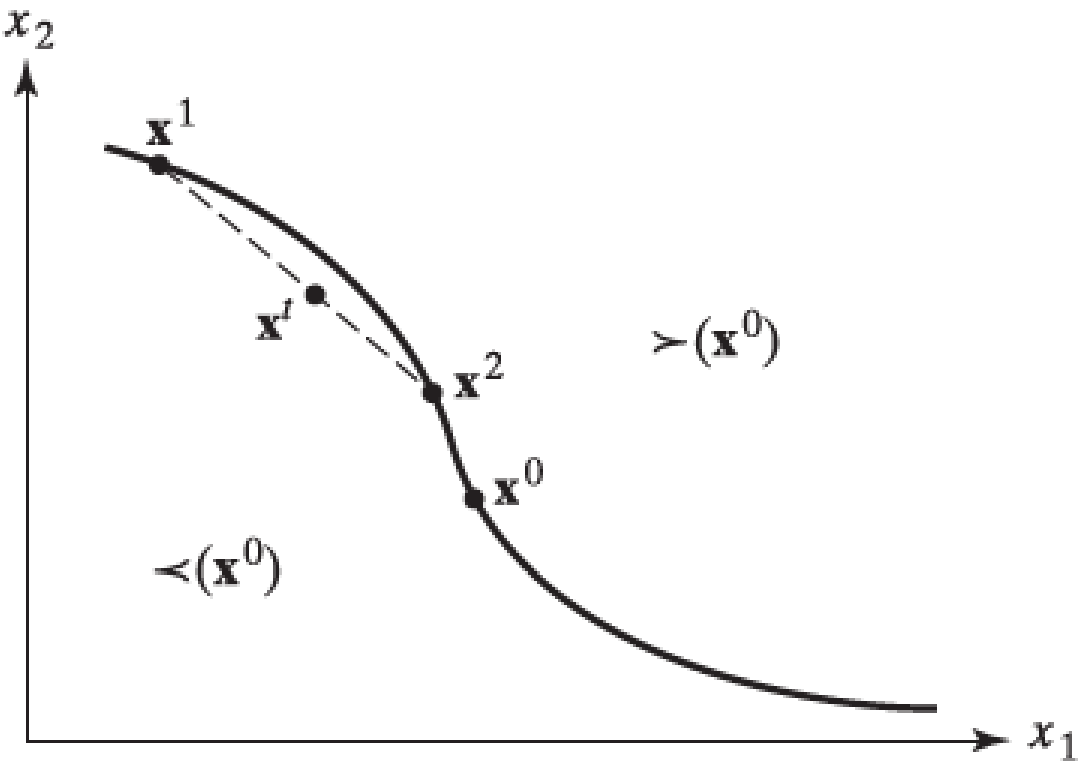
Preferences and utility

AXIOM 4′: Local Non-satiation. *For all* $\mathbf{x}^0 \in \mathbb{R}^n_+$, *and for all* $\varepsilon > 0$, *there exists some* $\mathbf{x} \in B_\varepsilon(\mathbf{x}^0) \cap \mathbb{R}^n_+$ *such that* $\mathbf{x} \succ \mathbf{x}^0$.



Preferences and utility

AXIOM 4: Strict Monotonicity. *For all* $\mathbf{x}^0, \mathbf{x}^1 \in \mathbb{R}^n_+$, *if* $\mathbf{x}^0 \geq \mathbf{x}^1$ *then* $\mathbf{x}^0 \succsim \mathbf{x}^1$, *while if* $\mathbf{x}^0 \gg \mathbf{x}^1$, *then* $\mathbf{x}^0 \succ \mathbf{x}^1$.

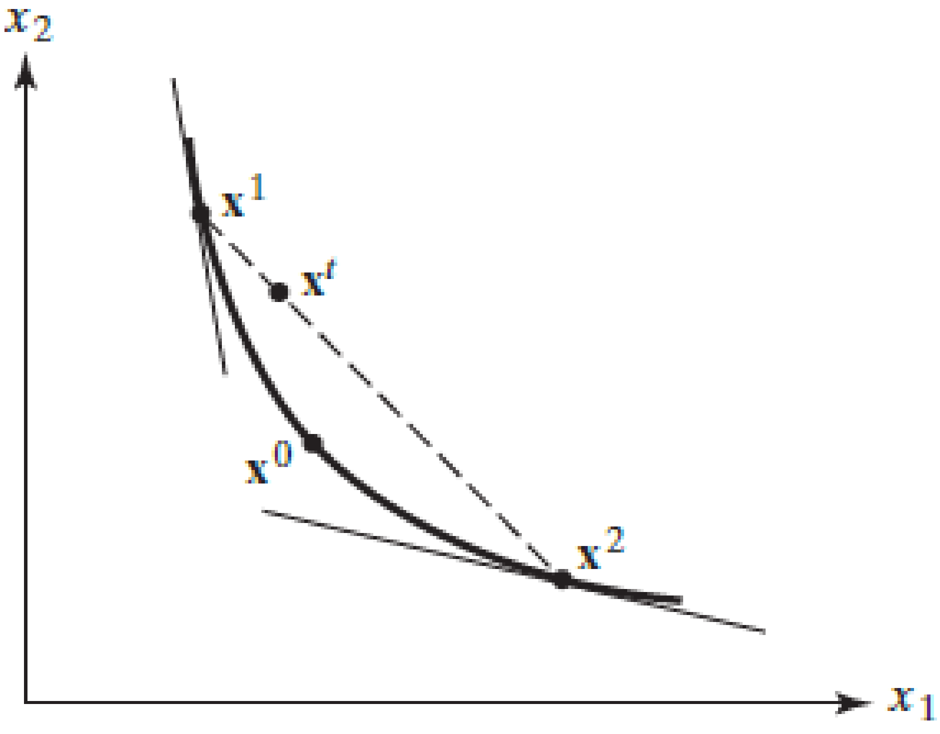


Preferences and utility

AXIOM 5′: Convexity. *If* $\mathbf{x}^1 \succsim \mathbf{x}^0$, *then* $t\mathbf{x}^1 + (1-t)\mathbf{x}^0 \succsim \mathbf{x}^0$ *for all* $t \in [0, 1]$.

A slightly stronger version of this is the following:

AXIOM 5: Strict Convexity. *If* $\mathbf{x}^1 \neq \mathbf{x}^0$ *and* $\mathbf{x}^1 \succsim \mathbf{x}^0$, *then* $t\mathbf{x}^1 + (1-t)\mathbf{x}^0 \succ \mathbf{x}^0$ *for all* $t \in (0, 1)$.



The utility function

A real-valued function $u: \mathbb{R}^n_+ \to \mathbb{R}$ *is called a utility function representing the preference relation* $\succsim$, *if for all* $\mathbf{x}^0, \mathbf{x}^1 \in \mathbb{R}^n_+$, $u(\mathbf{x}^0) \geq u(\mathbf{x}^1) \Longleftrightarrow \mathbf{x}^0 \succsim \mathbf{x}^1$.

If the binary relation $\succsim$ *is complete, transitive, continuous, and strictly monotonic, there exists a continuous real-valued function,* $u: \mathbb{R}^n_+ \to \mathbb{R}$, *which represents* $\succsim$.

Let $\succsim$ *be a preference relation on* $\mathbb{R}^n_+$ *and suppose* $u(\mathbf{x})$ *is a utility function that represents it. Then* $v(\mathbf{x})$ *also represents* $\succsim$ *if and only if* $v(\mathbf{x}) = f(u(\mathbf{x}))$ *for every* $\mathbf{x}$, *where* $f: \mathbb{R} \to \mathbb{R}$ *is strictly increasing on the set of values taken on by u.*

Let $\succsim$ *be represented by* $u: \mathbb{R}^n_+ \to \mathbb{R}$. *Then:*

1. $u(\mathbf{x})$ *is strictly increasing if and only if* $\succsim$ *is strictly monotonic.*
2. $u(\mathbf{x})$ *is quasiconcave if and only if* $\succsim$ *is convex.*
3. $u(\mathbf{x})$ *is strictly quasiconcave if and only if* $\succsim$ *is strictly convex.*

Consumer's problem

Find $\mathbf{x}^* \in B$ such that $\mathbf{x}^* \succsim \mathbf{x}$ for all $\mathbf{x} \in B$

Consumption bundle ($\mathbf{x}^*$)

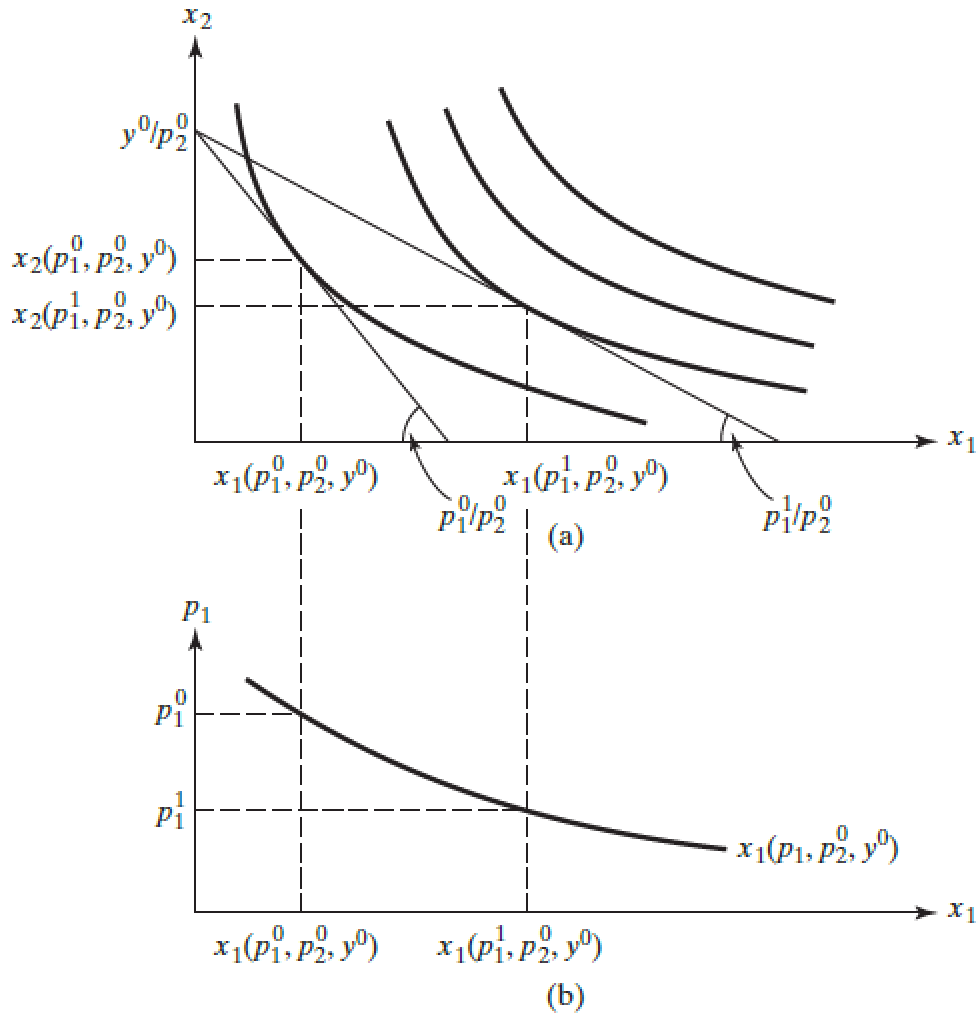
Feasible set (B)

Utility maximization problem

$$\max_{\mathbf{x} \in \mathbb{R}^n_+} u(\mathbf{x}) \qquad \text{s.t.} \qquad \mathbf{p} \cdot \mathbf{x} \leq y.$$

➔ Solution to the UMP is Marshallian demand functions

Consumer's problem and consumer demand behavior



The indirect utility function

Indirect utility function is a maximum value function:

$$v(\mathbf{p}, y) = \max_{\mathbf{x} \in \mathbb{R}^n_+} u(\mathbf{x}) \qquad \text{s.t.} \qquad \mathbf{p} \cdot \mathbf{x} \le y.$$

$$v(\mathbf{p}, y) = u(\mathbf{x}(\mathbf{p}, y))$$

Properties of the Indirect Utility Function

If $u(\mathbf{x})$ is continuous and strictly increasing on $\mathbb{R}^n_+$, then $v(\mathbf{p}, y)$ defined in (1.12) is

1. *Continuous on $\mathbb{R}^n_{++} \times \mathbb{R}_+$,*
2. *Homogeneous of degree zero in $(\mathbf{p}, y)$,*
3. *Strictly increasing in y,*
4. *Decreasing in $\mathbf{p}$,*
5. *Quasiconvex in $(\mathbf{p}, y)$.*

Moreover, it satisfies

6. *Roy's identity: If $v(\mathbf{p}, y)$ is differentiable at $(\mathbf{p}^0, y^0)$ and $\partial v(\mathbf{p}^0, y^0)/\partial y \neq 0$, then*

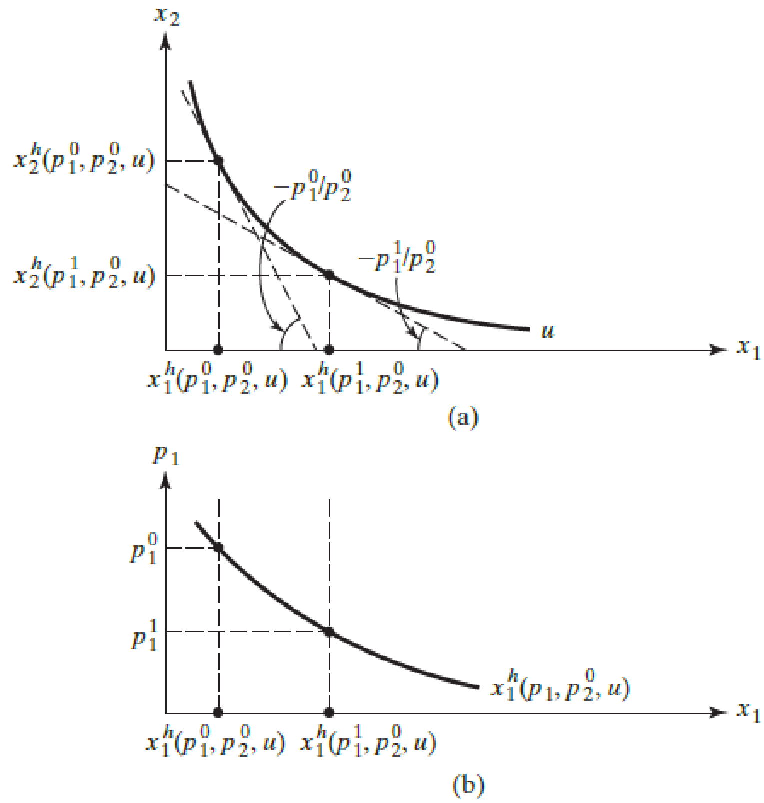
$$x_i(\mathbf{p}^0, y^0) = -\frac{\partial v(\mathbf{p}^0, y^0)/\partial p_i}{\partial v(\mathbf{p}^0, y^0)/\partial y}, \qquad i = 1, \ldots, n.$$

The expenditure function

$$e(\mathbf{p}, u) \equiv \min_{\mathbf{x} \in \mathbb{R}^n_+} \mathbf{p} \cdot \mathbf{x} \qquad \text{s.t.} \qquad u(\mathbf{x}) \geq u$$

$$e(\mathbf{p}, u) = \mathbf{p} \cdot \mathbf{x}^h(\mathbf{p}, u),$$

→ Hicksian (compensated) demand functions



Properties of the expenditure function

Properties of the Expenditure Function

If $u(\cdot)$ is continuous and strictly increasing, then $e(\mathbf{p}, u)$ defined in (1.14) is

1. *Zero when u takes on the lowest level of utility in $\mathcal{U}$,*
2. *Continuous on its domain $\mathbb{R}^n_{++} \times \mathcal{U}$,*
3. *For all $\mathbf{p} \gg \mathbf{0}$, strictly increasing and unbounded above in u,*
4. *Increasing in $\mathbf{p}$,*
5. *Homogeneous of degree 1 in $\mathbf{p}$,*
6. *Concave in $\mathbf{p}$.*

If, in addition, $u(\cdot)$ is strictly quasiconcave, we have

7. *Shephard's lemma: $e(\mathbf{p}, u)$ is differentiable in $\mathbf{p}$ at $(\mathbf{p}^0, u^0)$ with $\mathbf{p}^0 \gg \mathbf{0}$, and*

$$\frac{\partial e(\mathbf{p}^0, u^0)}{\partial p_i} = x_i^h(\mathbf{p}^0, u^0), \qquad i = 1, \ldots, n.$$

Relations between IUF and EF

Let $v(\mathbf{p}, y)$ *and* $e(\mathbf{p}, u)$ *be the indirect utility function and expenditure function for some consumer whose utility function is continuous and strictly increasing. Then for all* $\mathbf{p} \gg \mathbf{0}$, $y \geq 0$, *and* $u \in \mathcal{U}$*:*

1. $e(\mathbf{p}, v(\mathbf{p}, y)) = y.$
2. $v(\mathbf{p}, e(\mathbf{p}, u)) = u.$

$$e(\mathbf{p}, u) = v^{-1}(\mathbf{p} : u).$$

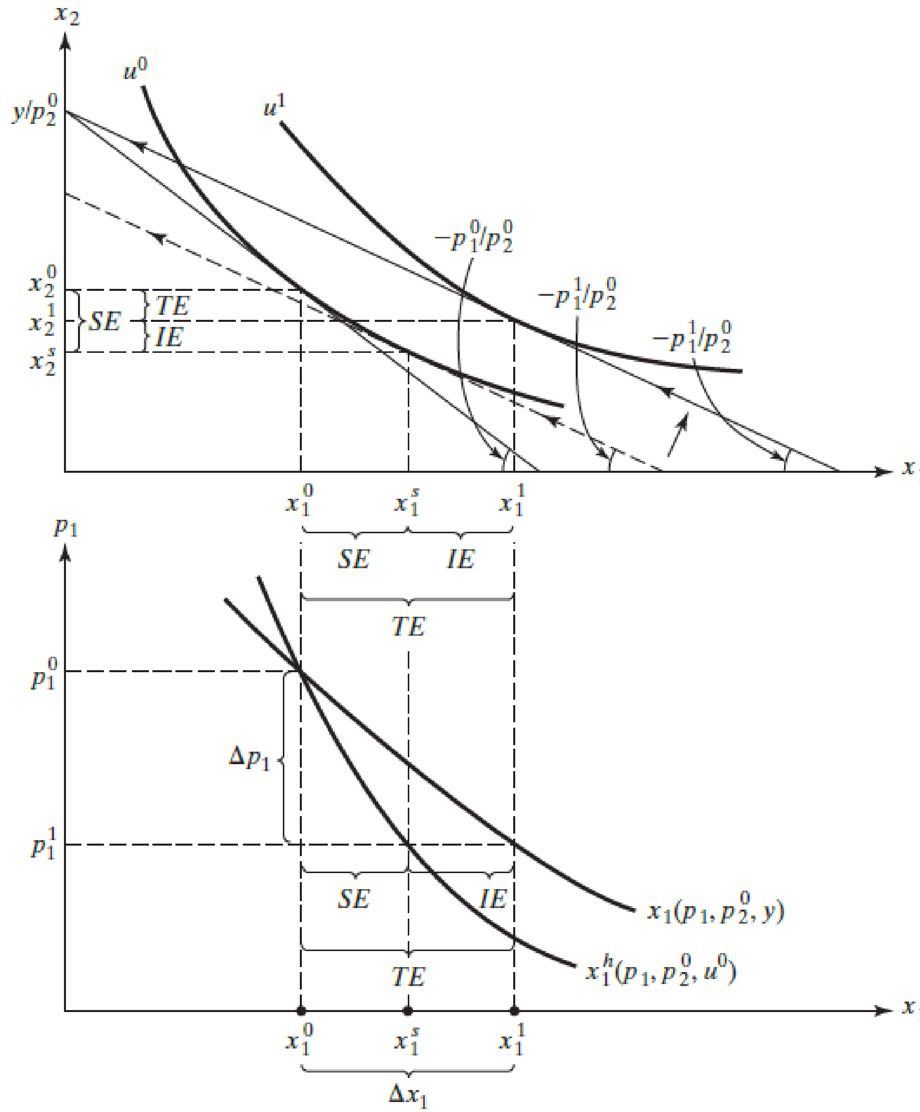
$$v(\mathbf{p}, y) = e^{-1}(\mathbf{p} : y).$$

Duality between Marshallian and Hicksian demand functions

Under Assumption 1.2 we have the following relations between the Hicksian and Marshallian demand functions for $\mathbf{p} \gg \mathbf{0}$, $y \geq 0$, $u \in \mathcal{U}$, *and* $i = 1, \ldots, n$:

1. $x_i(\mathbf{p}, y) = x_i^h(\mathbf{p}, v(\mathbf{p}, y))$.
2. $x_i^h(\mathbf{p}, u) = x_i(\mathbf{p}, e(\mathbf{p}, u))$.

Income and substitution effects



The Slutsky equation

Let $\mathbf{x}(\mathbf{p}, y)$ *be the consumer's Marshallian demand system. Let* u^* *be the level of utility the consumer achieves at prices* $\mathbf{p}$ *and income* y*. Then,*

$$\underbrace{\frac{\partial x_i(\mathbf{p}, y)}{\partial p_j}}_{TE} = \underbrace{\frac{\partial x_i^h(\mathbf{p}, u^*)}{\partial p_j}}_{SE} \underbrace{- x_j(\mathbf{p}, y)\frac{\partial x_i(\mathbf{p}, y)}{\partial y}}_{IE}, \qquad i, j = 1, \ldots, n.$$

Negative Own-Substitution Terms

Let $x_i^h(\mathbf{p}, u)$ *be the Hicksian demand for good* i*. Then*

$$\frac{\partial x_i^h(\mathbf{p}, u)}{\partial p_i} \leq 0, \qquad i = 1, \ldots, n.$$

The Law of Demand

A decrease in the own price of a normal good will cause quantity demanded to increase. If an own price decrease causes a decrease in quantity demanded, the good must be inferior.

Symmetric Substitution Terms

Let $\mathbf{x}^h(\mathbf{p}, u)$ *be the consumer's system of Hicksian demands and suppose that* $e(\cdot)$ *is twice continuously differentiable. Then,*

$$\frac{\partial x_i^h(\mathbf{p}, u)}{\partial p_j} = \frac{\partial x_j^h(\mathbf{p}, u)}{\partial p_i}, \qquad i, j = 1, \ldots, n.$$

Negative Semidefinite Substitution Matrix

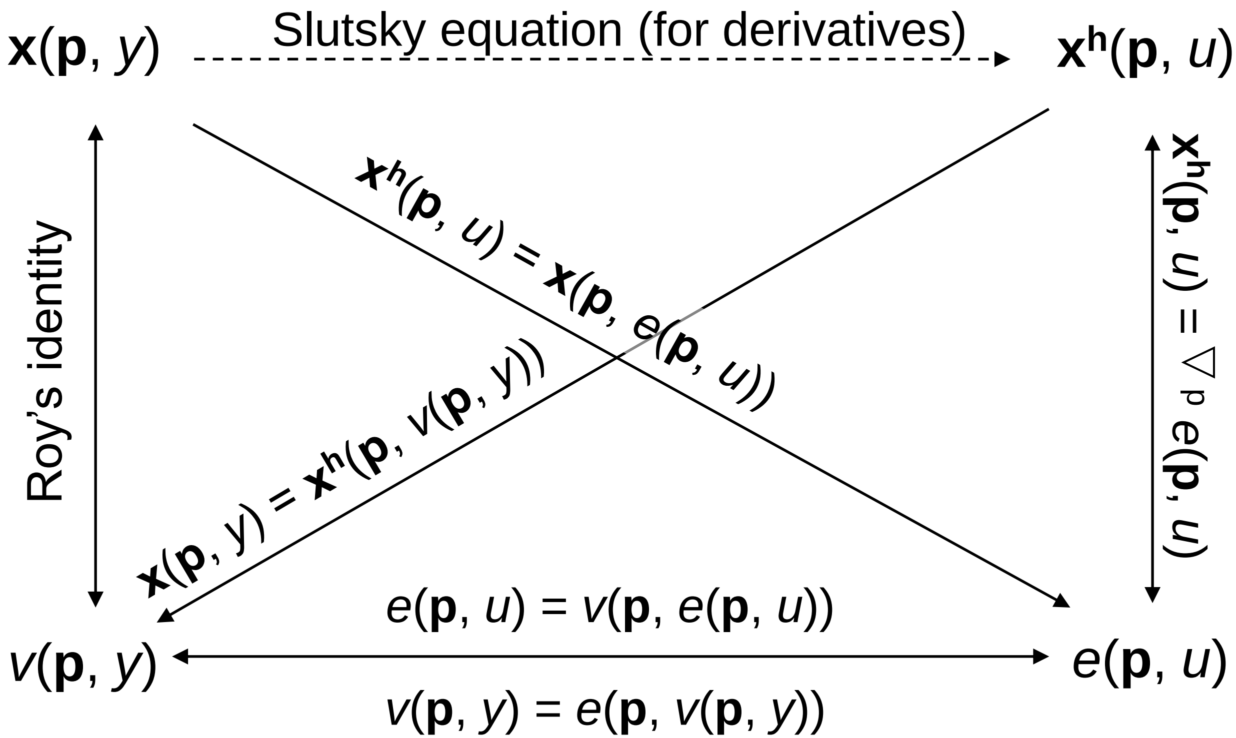
Let $\mathbf{x}^h(\mathbf{p}, u)$ *be the consumer's system of Hicksian demands, and let*

$$\sigma(\mathbf{p}, u) \equiv \begin{pmatrix} \frac{\partial x_1^h(\mathbf{p}, u)}{\partial p_1} & \cdots & \frac{\partial x_1^h(\mathbf{p}, u)}{\partial p_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial x_n^h(\mathbf{p}, u)}{\partial p_1} & \cdots & \frac{\partial x_n^h(\mathbf{p}, u)}{\partial p_n} \end{pmatrix},$$

called the **substitution matrix**, *contain all the Hicksian substitution terms. Then the matrix* $\sigma(\mathbf{p}, u)$ *is negative semidefinite.*

Relationships between UMP and EMP

Utility maximisation | Expenditure minimisation



Adapted from Mas-Collel, Whinston and Green (1995); p. 75

Theory of the Firm

Production

- Production possibility set
- Production plan
- Production function

Properties of the Production Function

The production function, $f\colon \mathbb{R}^n_+ \to \mathbb{R}_+$, is continuous, strictly increasing, and strictly quasiconcave on $\mathbb{R}^n_+$, and $f(\mathbf{0}) = 0$.

The Elasticity of Substitution

For a production function $f(\mathbf{x})$, the elasticity of substitution of input j for input i at the point $\mathbf{x}^0 \in \mathbb{R}^n_{++}$ is defined as

$$\sigma_{ij}(\mathbf{x}^0) \equiv \left(\frac{d \ln MRTS_{ij}(\mathbf{x}(r))}{d \ln r} \Bigg|_{r = x_j^0 / x_i^0} \right)^{-1},$$

where $\mathbf{x}(r)$ is the unique vector of inputs $\mathbf{x} = (x_1, \ldots, x_n)$ such that (i) $x_j/x_i = r$, (ii) $x_k = x_k^0$ for $k \neq i, j$, and (iii) $f(\mathbf{x}) = f(\mathbf{x}^0)$.[2]

Production

(Global) Returns to Scale

A production function $f(\mathbf{x})$ has the property of (globally):

1. *Constant returns to scale if $f(t\mathbf{x}) = tf(\mathbf{x})$ for all $t > 0$ and all $\mathbf{x}$;*
2. *Increasing returns to scale if $f(t\mathbf{x}) > tf(\mathbf{x})$ for all $t > 1$ and all $\mathbf{x}$;*
3. *Decreasing returns to scale if $f(t\mathbf{x}) < tf(\mathbf{x})$ for all $t > 1$ and all $\mathbf{x}$.*

(Local) Returns to Scale

The elasticity of scale at the point $\mathbf{x}$ is defined as

$$\mu(\mathbf{x}) \equiv \lim_{t \to 1} \frac{d \ln[f(t\mathbf{x})]}{d \ln(t)} = \frac{\sum_{i=1}^{n} f_i(\mathbf{x}) x_i}{f(\mathbf{x})}.$$

Returns to scale are locally constant, increasing, or decreasing as $\mu(\mathbf{x})$ is equal to, greater than, or less than one. The elasticity of scale and the output elasticities of the inputs are related as follows:

$$\mu(\mathbf{x}) = \sum_{i=1}^{n} \mu_i(\mathbf{x}).$$

Cost

The Cost Function

The cost function, defined for all input prices $\mathbf{w} \gg \mathbf{0}$ *and all output levels* $y \in f(\mathbb{R}^n_+)$ *is the minimum-value function,*

$$c(\mathbf{w}, y) \equiv \min_{\mathbf{x} \in \mathbb{R}^n_+} \mathbf{w} \cdot \mathbf{x} \qquad s.t. \qquad f(\mathbf{x}) \geq y.$$

If $\mathbf{x}(\mathbf{w}, y)$ *solves the cost-minimisation problem, then*

$$c(\mathbf{w}, y) = \mathbf{w} \cdot \mathbf{x}(\mathbf{w}, y).$$

→ Conditional input demand

Properties of the cost function

If f is continuous and strictly increasing, then $c(\mathbf{w}, y)$ is

1. *Zero when $y = 0$,*
2. *Continuous on its domain,*
3. *For all $\mathbf{w} \gg 0$, strictly increasing and unbounded above in y,*
4. *Increasing in $\mathbf{w}$,*
5. *Homogeneous of degree one in $\mathbf{w}$,*
6. *Concave in $\mathbf{w}$.*

Moreover, if f is strictly quasiconcave we have

7. *Shephard's lemma: $c(\mathbf{w}, y)$ is differentiable in $\mathbf{w}$ at $(\mathbf{w}^0, y^0)$ whenever $\mathbf{w}^0 \gg 0$, and*

$$\frac{\partial c(\mathbf{w}^0, y^0)}{\partial w_i} = x_i(\mathbf{w}^0, y^0), \qquad i = 1, \ldots, n.$$

Cost and Conditional Input Demands when Production is Homothetic

1. *When the production function satisfies Assumption 3.1 and is homothetic,*

   (a) *the cost function is multiplicatively separable in input prices and output and can be written* $c(\mathbf{w}, y) = h(y)c(\mathbf{w}, 1)$, *where* $h(y)$ *is strictly increasing and* $c(\mathbf{w}, 1)$ *is the unit cost function, or the cost of 1 unit of output;*

   (b) *the conditional input demands are multiplicatively separable in input prices and output and can be written* $\mathbf{x}(\mathbf{w}, y) = h(y)\mathbf{x}(\mathbf{w}, 1)$, *where* $h'(y) > 0$ *and* $\mathbf{x}(\mathbf{w}, 1)$ *is the conditional input demand for 1 unit of output.*

2. *When the production function is homogeneous of degree* $\alpha > 0$,

   (a) $c(\mathbf{w}, y) = y^{1/\alpha}c(\mathbf{w}, 1)$;

   (b) $\mathbf{x}(\mathbf{w}, y) = y^{1/\alpha}\mathbf{x}(\mathbf{w}, 1)$.

The Short-Run, or Restricted, Cost Function

Let the production function be $f(\mathbf{z})$, *where* $\mathbf{z} \equiv (\mathbf{x}, \bar{\mathbf{x}})$. *Suppose that* $\mathbf{x}$ *is a subvector of variable inputs and* $\bar{\mathbf{x}}$ *is a subvector of fixed inputs. Let* $\mathbf{w}$ *and* $\bar{\mathbf{w}}$ *be the associated input prices for the variable and fixed inputs, respectively. The short-run, or restricted, total cost function is defined as*

$$sc(\mathbf{w}, \bar{\mathbf{w}}, y; \bar{\mathbf{x}}) \equiv \min_{\mathbf{x}} \ \mathbf{w} \cdot \mathbf{x} + \bar{\mathbf{w}} \cdot \bar{\mathbf{x}} \qquad s.t. \qquad f(\mathbf{x}, \bar{\mathbf{x}}) \geq y.$$

If $\mathbf{x}(\mathbf{w}, \bar{\mathbf{w}}, y; \bar{\mathbf{x}})$ *solves this minimisation problem, then*

$$sc(\mathbf{w}, \bar{\mathbf{w}}, y; \bar{\mathbf{x}}) = \mathbf{w} \cdot \mathbf{x}(\mathbf{w}, \bar{\mathbf{w}}, y; \bar{\mathbf{x}}) + \bar{\mathbf{w}} \cdot \bar{\mathbf{x}}.$$

The optimised cost of the variable inputs, $\mathbf{w} \cdot \mathbf{x}(\mathbf{w}, \bar{\mathbf{w}}, y; \bar{\mathbf{x}})$, *is called* **total variable cost**. *The cost of the fixed inputs,* $\bar{\mathbf{w}} \cdot \bar{\mathbf{x}}$, *is called* **total fixed cost**.

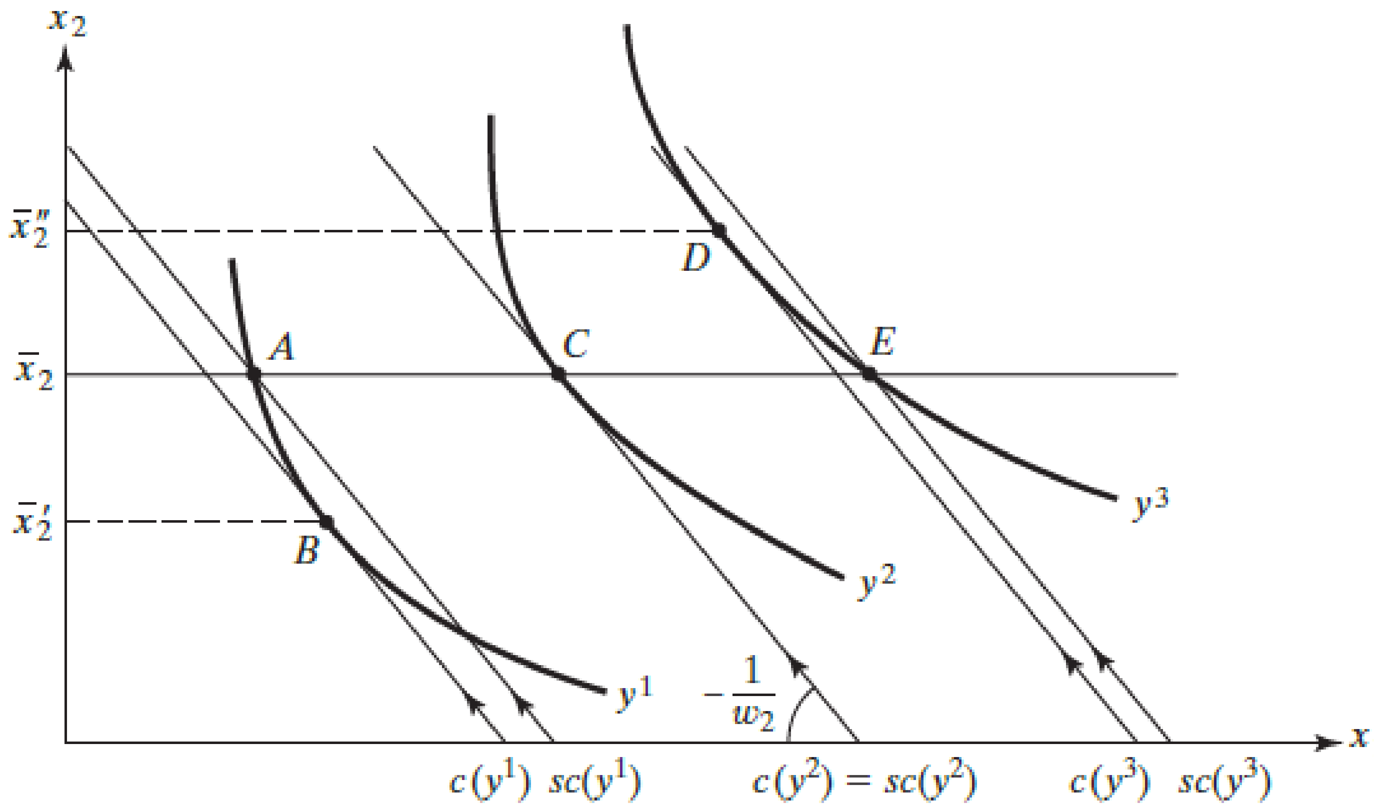


Figure 3.5. $sc(\mathbf{w}, \bar{\mathbf{w}}, y; \bar{\mathbf{x}}) \geq c(\mathbf{w}, \bar{\mathbf{w}}, y)$ for all output levels y.

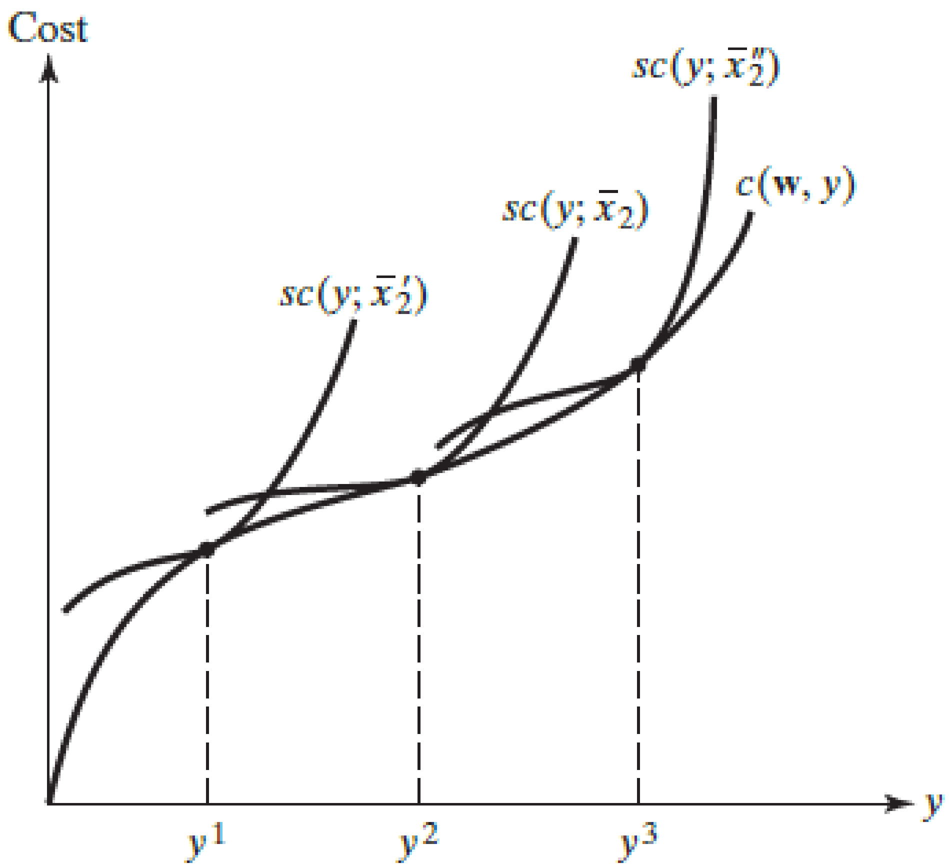


Figure 3.6. Long-run total cost is the envelope of short-run total cost.

The competitive firm

Profit maximization

REVENUE

$$\max_{(\mathbf{x},y)\geq 0} \; py - \mathbf{w}\cdot\mathbf{x} \qquad \text{s.t.} \qquad f(\mathbf{x}) \geq y,$$

COST

$f(x) = y$

...or

$$\max_{y\geq 0} \; py - c(\mathbf{w}, y).$$

→ Cost function

$\pi = p \cdot f(\vec{x}) - \vec{w}\,\vec{x}$

$x_1^*(p, \vec{w})$

$x_2^*(p, \vec{w})$

The profit function

The firm's profit function depends only on input and output prices and is defined as the maximum-value function,

$$\pi(p, \mathbf{w}) \equiv \max_{(\mathbf{x},y)\geq \mathbf{0}} py - \mathbf{w}\cdot\mathbf{x} \quad \text{s.t.} \quad f(\mathbf{x}) \geq y.$$

Properties of the Profit Function

If f satisfies Assumption 3.1, then for $p \geq 0$ and $\mathbf{w} \geq \mathbf{0}$, the profit function $\pi(p, \mathbf{w})$, where well-defined, is continuous and

1. *Increasing in p,*
2. *Decreasing in $\mathbf{w}$,*
3. *Homogeneous of degree one in $(p, \mathbf{w})$,*
4. *Convex in $(p, \mathbf{w})$,*
5. *Differentiable in $(p, \mathbf{w}) \gg \mathbf{0}$. Moreover, under the additional assumption that f is strictly concave (Hotelling's lemma),*

$$\frac{\partial \pi(p, \mathbf{w})}{\partial p} = y(p, \mathbf{w}), \quad \text{and} \quad \frac{-\partial \pi(p, \mathbf{w})}{\partial w_i} = x_i(p, \mathbf{w}), \quad i = 1, 2, \ldots, n.$$

OUTPUT SUPPLY

Properties of Output Supply and Input Demand Functions

Suppose that f is a strictly concave production function satisfying Assumption 3.1 and that its associated profit function, $\pi(\mathbf{p}, y)$, is twice continuously differentiable. Then, for all $p > 0$ and $\mathbf{w} \gg \mathbf{0}$ where it is well defined:

1. *Homogeneity of degree zero:*

$$y(tp, t\mathbf{w}) = y(p, \mathbf{w}) \quad \text{for all} \quad t > 0,$$
$$x_i(tp, t\mathbf{w}) = x_i(p, \mathbf{w}) \quad \text{for all} \quad t > 0 \quad \text{and} \quad i = 1, \ldots, n.$$

2. *Own-price effects:*[3]

$$\frac{\partial y(p, \mathbf{w})}{\partial p} \geq 0,$$
$$\frac{\partial x_i(p, \mathbf{w})}{\partial w_i} \leq 0 \quad \text{for all} \quad i = 1, \ldots, n.$$

3. *The substitution matrix*

$$\begin{pmatrix} \frac{\partial y(p, \mathbf{w})}{\partial p} & \frac{\partial y(p, \mathbf{w})}{\partial w_1} & \cdots & \frac{\partial y(p, \mathbf{w})}{\partial w_n} \\ \frac{-\partial x_1(p, \mathbf{w})}{\partial p} & \frac{-\partial x_1(p, \mathbf{w})}{\partial w_1} & \cdots & \frac{-\partial x_1(p, \mathbf{w})}{\partial w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{-\partial x_n(p, \mathbf{w})}{\partial p} & \frac{-\partial x_n(p, \mathbf{w})}{\partial w_1} & \cdots & \frac{-\partial x_n(p, \mathbf{w})}{\partial w_n} \end{pmatrix}$$

is symmetric and positive semidefinite.

Ex $q = \sqrt{x_1} + \sqrt{x_2} = f(x_1, x_2)$

$\sqrt{x_1} = x_1^{\frac{1}{2}}$

$$\min C = w_1 x_1 + w_2 x_2$$

$$\text{s.t.} \quad \sqrt{x_1} + \sqrt{x_2} = q$$

$$\mathcal{L} = w_1 x_1 + w_2 x_2 - \lambda\left(\sqrt{x_1} + \sqrt{x_2} - q\right)$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = w_1 - \frac{1}{2}\lambda \cdot x_1^{-\frac{1}{2}} = 0 \Rightarrow w_1 = \frac{1}{2}\lambda x_1^{-\frac{1}{2}}$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = w_2 - \frac{1}{2}\lambda x_2^{-\frac{1}{2}} = 0 \Rightarrow w_2 = \frac{1}{2}\lambda x_2^{-\frac{1}{2}}$$

$$\frac{w_1}{w_2} = \frac{\frac{1}{2}\lambda x_1^{-\frac{1}{2}}}{\frac{1}{2}\lambda x_2^{-\frac{1}{2}}} \Rightarrow \frac{w_1}{w_2} = \frac{x_2^{\frac{1}{2}}}{x_1^{\frac{1}{2}}} = \sqrt{\frac{x_2}{x_1}}$$

$$\frac{w_1^2}{w_2^2} = \frac{x_2}{x_1}$$

$$\boxed{x_2 = \frac{w_1^2}{w_2^2} \cdot x_1}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = -\left(\sqrt{x_1} + \sqrt{x_2} - q\right) = 0$$

$$\sqrt{x_1} + \sqrt{x_1} \cdot \frac{w_1}{w_2} = q$$

$$\sqrt{x_1} + \sqrt{x_1}\,\frac{w_1}{w_2} = q \qquad x_1^*(w_1, w_2, q)$$

$$\sqrt{x_1}\left(1 + \frac{w_1}{w_2}\right) = q$$

$$\sqrt{x_1}\;\frac{w_1 + w_2}{w_2} = q$$

$$\sqrt{x_1} = \frac{w_2\, q}{w_1 + w_2}$$

$$x_1^* = \left(\frac{w_2\, q}{w_1 + w_2}\right)^2 \; ; \quad x_2^* = \left(\frac{w_1\, q}{w_1 + w_2}\right)^2$$

COST FUNCTION

$$C^* = w_1 x_1^* + w_2 x_2^* = w_1 \cdot \left(\frac{w_2 q}{w_1 + w_2}\right)^2 + w_2 \left(\frac{w_1 q}{w_1 + w_2}\right)^2$$

$$= \left(\frac{q}{w_1 + w_2}\right)^2 \left(w_1 w_2^2 + w_1^2 w_2 \right) =$$

$$= \left(\frac{q}{w_1 + w_2}\right)^2 w_1 w_2 \left(w_1 + w_2\right) = \boxed{\frac{w_1 w_2}{w_1 + w_2} q^2}$$

$$\max \Pi = P \cdot \left(\sqrt{x_1} + \sqrt{x_2}\right) - w_1 x_1 - w_2 x_2$$

$$\frac{\partial \Pi}{\partial x_1} = \frac{1}{2} P \cdot x_1^{-\frac{1}{2}} - w_1 = 0 \Rightarrow \frac{1}{2} \frac{P}{\sqrt{x_1}} = w_1 \Rightarrow x_1^* = \frac{P^2}{4 w_1^2}$$

$$\frac{\partial \Pi}{\partial x_2} = \frac{1}{2} P \cdot x_2^{-\frac{1}{2}} - w_2 = 0 \Rightarrow \quad x_2^* = \frac{P^2}{4 w_2^2}$$

$$\left.\begin{array}{l} \frac{1}{2} P x_1^{-\frac{1}{2}} = w_1 \\ \frac{1}{2} P x_2^{-\frac{1}{2}} = w_2 \end{array}\right\} \quad \frac{x_2^{\frac{1}{2}}}{x_1^{\frac{1}{2}}} = \frac{w_1}{w_2} = \boxed{\sqrt{\frac{x_2}{x_1}} = \frac{w_1}{w_2}}$$

$$q^* = \sqrt{x_1^*} + \sqrt{x_2^*} = \sqrt{\frac{p^2}{4w_1^2}} + \sqrt{\frac{p^2}{4w_2^2}} = \frac{p}{2w_1} + \frac{p}{2w_2}$$

$$= \frac{p}{2}\left(\frac{1}{w_1} + \frac{1}{w_2}\right) = \frac{p}{2}\left(\frac{w_1 + w_2}{w_1 w_2}\right)$$

$$\Pi^* = p \cdot q^* - w_1 \cdot \frac{p^2}{4w_1^2} - w_2 \cdot \frac{p^2}{4w_2^2}$$

$$= \frac{2p^2 (w_1 + w_2)}{4 w_1 w_2} - \frac{p^2}{4w_1} - \frac{p^2}{4w_2} = \frac{p^2 (w_1 + w_2)}{4 w_1 w_2}$$

$$- \frac{p^2}{4}\left(\frac{w_1 + w_2}{w_1 w_2}\right)$$

$$\pi^* = \frac{p^2(w_1+w_2)}{4w_1w_2}$$

$$q^* = y^* = \frac{\partial \pi^*}{\partial p} = \frac{2p(w_1+w_2)}{4w_1w_2} = \boxed{\frac{p(w_1+w_2)}{2w_1w_2}}$$

$$x_1^* = -\frac{\partial \pi}{\partial w_1} = -\left[\frac{p^2 \cdot 4w_1w_2 - p^2(w_1+w_2)\cdot 4w_2}{4w_1^2w_2^2}\right]$$

$$-\frac{p^2w_1w_2 - p^2w_1w_2 - p^2w_2^2}{4w_1^2w_2^2} = \frac{p^2w_2^2}{4w_1^2w_2^2} = \boxed{\frac{p^2}{4w_1^2}}$$

$$\max \pi = P f(x) - wx$$

$$f_{xx} = \frac{\partial f_x}{\partial x}$$

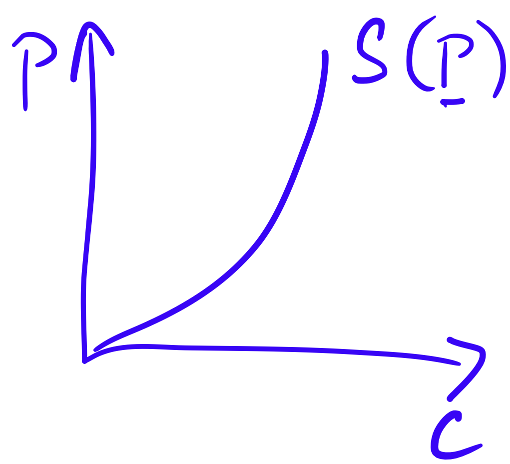
$$\frac{d\pi}{dx} = P \cdot \frac{df}{dx} - w = 0$$

$$\frac{df}{dx} = f_x$$

$$P \cdot f_x = w$$

$$f_x = \frac{w}{P}$$

$$q = A L^{\varepsilon} \Rightarrow L^* = \left(\frac{q}{A}\right)^{\frac{1}{\varepsilon}}$$



$$\varepsilon \xrightarrow{2} \eta_S$$

$$\eta = \frac{dS}{dP} \cdot \frac{P}{S}$$

$$\boxed{MC = P}$$

$$\frac{1}{\varepsilon} \cdot \frac{w}{B} \cdot q^{\frac{1}{\varepsilon}-1} = P$$

$$TC = w \cdot L = w\left(\frac{q}{A}\right)^{\frac{1}{\varepsilon}}$$

$$MC = \frac{dTC}{dq} = \frac{1}{\varepsilon} \cdot w \cdot \frac{q^{\frac{1}{\varepsilon}-1}}{A^{\frac{1}{\varepsilon}}}$$

$$q^{\frac{1-\varepsilon}{\varepsilon}} = \frac{\varepsilon \cdot P \cdot B}{w}$$

$$\boxed{q = \left(\frac{\varepsilon \cdot P \cdot B}{w}\right)^{\frac{\varepsilon}{1-\varepsilon}}}$$

$$q = \left(\frac{\varepsilon \cdot B \cdot P}{w} \right)^{\frac{\varepsilon}{1-\varepsilon}}$$

$$\frac{dq}{dP} = \frac{\varepsilon}{1-\varepsilon} P^{\frac{\varepsilon}{1-\varepsilon} - 1} \cdot \left(\frac{\varepsilon B}{w} \right)^{\frac{\varepsilon}{1-\varepsilon}}$$

$$\eta = \frac{dq}{dP} \cdot \frac{P}{q} = \frac{\varepsilon}{1-\varepsilon} P^{\frac{\varepsilon}{1-\varepsilon} - 1} \left(\frac{\varepsilon B}{w} \right)^{\frac{\varepsilon}{1-\varepsilon}} \cdot \frac{P^{1}}{\left(\frac{\varepsilon B}{w} \right)^{\frac{\varepsilon}{1-\varepsilon}}}$$

$$= \frac{\varepsilon}{1-\varepsilon} \cdot \frac{P^{\frac{\varepsilon}{1-\varepsilon} - 1 + 1}}{P^{\frac{\varepsilon}{1-\varepsilon}}} = \frac{\varepsilon}{1-\varepsilon}$$