The Acid Rain Game: a Formal and Mathematically Rigorous Analysis
CORRECTIONS AND SUPPLEMENTS

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Corrections:

1. The word 'Pareto-efficient' should be everywhere replaced by 'Pareto efficient' and the word 'Pareto-inefficient' by 'Pareto inefficient'.
2. At some places brackets '[ ]' are used (due to the publisher) after a function symbol; this is not nice, they should be replaced by '( )'. For instance this is two times the case in the first line of page 154.
3. *Page 140, Definition 1, property 7:* $\cdots$ and $\theta^j : (0, M^j] \to \mathbb{R}$ is twice
4. *Page 142, line 5* $\downarrow$*:* $\cdots$ $(Q^j \in (0, r^j])$,
5. *Page 142, line 5* $\downarrow$*:* $\cdots$ $(x^j \in (0, M^j])$ and $\cdots$
6. *Page 143, line 12* $\uparrow$*:* $\cdots$ and $B^j(\mathbf{z}) = \mathcal{B}^j(Q^j_\star(\mathbf{z}))$,
7. *Page 143, line 2* $\uparrow$*:*

$$\mathcal{B}^{j\prime}(c) = -\frac{T_{jj}^2 \mathcal{D}^{j\prime\prime}(T_{jj}(\mathcal{B}^j(c) + c))}{T_{jj}^2 \mathcal{D}^{j\prime\prime}(T_{jj}(\mathcal{B}^j(c) + c)) - \theta^{j\prime\prime}(\mathcal{B}^j(c))}. \tag{9.1}$$

8. *Page 143, line 1* $\uparrow$*:* $\cdots$ that $\mathcal{B}^{j\prime} \leq 0$ and $\cdots$
9. *page 144, line 9* $\uparrow$*:* $\cdots$ dominant emission level. See von Mouche (2004) for an example. [[See [1] below for this reference.]]
10. *Page 147, line 6* $\uparrow$*:* $\theta^{1\prime}(y^j) = \frac{T_{jj}}{\beta_{j1} T_{11}} \sum_{r=1}^N \cdots$
11. *Page 151, line 9* $\downarrow$*:* $\cdots$ each emission vector in $(\mathbf{a}, \mathbf{b}]$.
12. *Page 152, Research problem 2:* $\cdots$ than or equal to that in $\mathbf{n}$. (Compare Theorem 19.)
13. *Page 153:* $\cdots$ of the function $(h^j_c)' : (0, M^j)$
14. *Page 154, line 6* $\downarrow$*:* we have $\theta^{j\prime}(n^j) = T_j j \mathcal{D}^{j\prime}(Q^j(\mathbf{n}) \geq T_{jj} \mathcal{D}^{j\prime}(Q^j(\mathbf{z}))$
15. *Page 154, line 7* $\downarrow$*:* $\cdots$ thus $\theta^{j\prime}{}_-(n^j) \geq \theta^{j\prime}{}_+(z^j)$; because
16. *Page 154, line 11* $\downarrow$*:* $\cdots$ and thus that $(a, \ldots, a)$ is
17. *Page 155, in Proof of Theorem 11, line 6* $\downarrow$*:* that $(g^j_{\mathbf{z}^j})'(z^j) \geq 0$. From $\cdots$
18. *Page 155, in Proof of Theorem 11:* Replace everywhere '$\mathbf{z}\hat{j}$' by '$\mathbf{z}^{\hat{j}}$'.

19. *Page 155, line 6 ↑:* Take $k \in \text{supp}(\boldsymbol{\lambda})$ with $\cdots$

20. *Page 156, line 5 ↓:* $\cdots$ defined by $c^k := b^k$ $(k \neq \pi(N)) \cdots$

21. *Page 155, formula (9.5):* $\cdots \leq \frac{T_{ji}}{T_{jj}} \frac{1}{1-\theta^{j\prime\prime}/(T_{jj}^2 \mathcal{D}^{j\prime\prime})} \leq |D_i B^j|.$ (9.5)

22. *Page 158, line 14 ↓:* $\cdots$ Because $S^{j\prime} : [0,$

23. *Page 159, Note 3, line 3 ↓:* $\cdots$ in Welsch is almost the same as ours

24. *Page 160, note 15:* $\cdots$ for all permutations π of $\mathcal{N}$, $\mathbf{x} \in \mathbf{X} \cdots$

Comments: Concerning the 8 research problems on page 152. Problems 3 and 8 have been done in [1]. And in [1] it has been proved that the answer to Problem 4 is 'yes'.

Further reading:

[1] 2009. P. v. Mouche. Non-differentiability of Payoff Functions and Non-uniqueness of Nash Equilibria. World Academy of Science, Engineering and Technology, 53, 731–736.

[2] 2011. P. v. Mouche. On Games with Constant Nash Sum. In: Contributions to Game Theory and Management. Volume IV, 294-310. Editors: L. Petrosjan, N. Zenkevich. Graduate School of Management St. Petersburg. ISBN 978-5-9924-0069-4.

[3] 2014. M. Finus, P. v. Mouche and B. Rundshagen. On Uniqueness of Coalitional Equilibria. In: Contributions to Game Theory and Management. Volume VII, 51-60. Editors: L. Petrosjan, N. Zenkevich. St. Petersburg State University. ISSN 2310-2608.

[4] 2015. H. Folmer and P. v. Mouche. Nash Equilibria of Transboundary Pollution Games. In: Handbook of Research Methods and Applications in Environmental Studies, 504–524. Edward-Elgar. Editor: M. Ruth.

If you think that some other things should be added here, please let me know.