A predicting indicator for exchange rate instabilities.

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Abstract

Vehement international capital flows may make exchange rates unstable. Crises will occur. With a feedback system approach, using Fourier spectra, a criterion for instability is found. A first order and a second order system are treated. Also the correlation time of the flow may be indicative. The width of the power density spectrum should be tested as an indicator for the stability of exchange rates.

key words: exchange rate, dynamic system, Fourier spectrum, capital flow, indicator

Introduction

One of the most feared phenomena in economic life is an exchange rate crisis, a sudden collapse of the value of the currency of a country. If foreign investors lose their confidence, some of them will withdraw. Then the incoming capital flow diminishes, so that foreign money becomes rare and therefore expensive. This means that the home currency loses its value for foreign parties. Then more foreign investors will withdraw, so that the incoming capital flow decreases further and the home currency falls more and more. The end of this scenario is a collapse of the exchange rate and a long term disturbance of the capital market and the investments. Usually a long time before a crisis the balance of payments will show a shortage of export with respect to the import, so that the excess of incoming capital with respect to outgoing capital supplies the foreign money that is needed. The country that spends above its station will easily be at risk. In the same way an excess of export will be safe, as the country itself will be able to manage its exchange rate by controlling the flow of the outgoing investments.

Of course there are more factors. Stabilizing influences may be the confidence by tradition, the control by central banks of the capital flows with buffer stocks and the coordination by the IMF. Wage demands, inflation and other matters may affect the confidence and cause a risk. A country may strive after a stronger growth as a long term policy, it may promote its export or tax the import, or manage its inflation policy. All these instruments may influence the stability of the exchange rate. But predicting exchange rate crises on all these grounds may remain a precarious thing. So an indicator for the stability of a currency would be rather useful. Provided that an indicator does not influence the system it describes, it may warn and enable responsible authorities to take action. As an indicator works self fulfilling however, one should be careful not to undermine the stability of the system by its own influence.

After all this we raise our central question: how to construct a number that indicates how stable the exchange rate is. We will try to find an indicator on the basis of general principles of system dynamics. The capital flows that cause the risk of a collapse may be described with a dynamic system, that may be more or less stable. Of course political decisions will never be described by any dynamic system, as they concern the expression of a will. But a market, with the decisions of a multitude of people, may very well behave like a dynamic system with its own features of stability.

Starting points

We begin by considering the behavior of the international capital flows as a Markov process: the change in the system is a function of its actual situation only. The history does not matter. No deeper dynamics will be supposed. So only first order derivatives may be involved. If one is interested in a quantity y that is a small fluctuation with respect to a large and constant background, the system we look for may be linearized. As a first approach we propose the equation

$$dy / dt = p \cdot (u - y) . \qquad (1)$$

Suppose that a fly walks over a computer screen horizontally with some random velocity. Its position is u. With the mouse and cursor at position y we try to follow it. We see the deviation (u-y) and give the cursor a velocity dy/dt that is proportional to the deviation. The constant p is a real number. The problem described is a feedback system: the reaction dy/dt is adjusted on the basis of the observation of (u-y). In the same way as the walking fly disturbs our distance, the financial market is a system that is disturbed day after day by economic messages, by political news, by rumors and so on. So for international investors there arises every day a changed situation which causes them to react with an action dy/dt , which is stronger as their desired situation (u) deviates more from their actual situation (y). They strive to diminish the difference between the actual situation and that what they have in mind. Now we define y more precisely. Let Y be the net capital flow to a country in one day (incoming minus outgoing). We consider only the capital flow as generated by a free market. The interventions by central banks or the IMF are kept out of consideration, because they are governed by political decisions. Of course these interventions may stabilize exchange rates. But as soon as the market becomes unstable, which means that any small cause may lead to a disruption of the capital flows, the central banks are lost. Then their buffer stocks will be insufficient to prevent a crisis. Our system dynamics approach should deal with the behavior of the market. We denote the average value of Y over a long period as $\langle Y \rangle$. The fluctuations are now given by $y = Y - \langle Y \rangle$. The disturbance u may have all kinds of causes. We let these an open question. With these starting points we may formulate our central problem: is it possible to find a criterion that determines how far the market is stable or how far there is the risk of a crisis?

Dynamic behavior of (1)

We suppose that the system is brought out of equilibrium by an initial step u, from u=1 at $t<0$ to u=0 at $t>0$. Then y begins in t=0 with y(0)=1. We get $dy/dt = -p.y$. The solution is $y=\exp(-p.t)$. This solution is stable for $p>0$ and unstable for $p<0$. The critical value p=0 marks the transition from a stable situation to an unstable one.

Now we use Fourier integrals (in all the integrals the range will be from $-\infty$ to $+\infty$):

$$\mathbf{Y}(\omega) = \int y(t) \exp(i.\omega.t)\, dt \ , \qquad (2)$$

and

$$\mathbf{U}(\omega) = \int u(t) \exp(i.\omega.t)\, dt \ . \qquad (3)$$

The theory of Fourier integrals [ref. Butkov, p. 260-262] states that

$$y(t) = (1/2\pi) \int \mathbf{Y}(\omega) \exp(-i.\omega.t)\, d\omega \ , \qquad (4)$$

and

$$u(t) = (1/2\pi) \int \mathbf{U}(\omega) \exp(-i.\omega.t)\, d\omega \ . \qquad (5)$$

We substitute (4) and (5) in (1) and obtain

$$\int [(-i.\omega+p).\mathbf{Y} - p.\mathbf{U}] \exp(-i.\omega.t)\, d\omega = 0 \ , \qquad (6)$$

so that

$$\mathbf{Y} = \mathbf{U}\ p / (p-i.\omega) \ . \qquad (7)$$

We pass to the power density spectrum $\mathbf{Y}(\omega).\mathbf{Y}^*(\omega)$ with

$$\mathbf{Y}.\mathbf{Y}^* = \mathbf{U}.\mathbf{U}^*\ p^2 / (p^2 + \omega^2) \ . \qquad (8)$$

Here $\mathbf{Y}^*$ denotes the complex conjugate of $\mathbf{Y}$. We do not make any assumption on the causes of the disturbance u but that the stream of incidents and rumors leads to white noise:

$$\mathbf{U}.\mathbf{U}^* = a^2 \ , \qquad (9)$$

where a is a constant, real number. Now we know the power density spectrum of y(t) by

$$\mathbf{Y}.\mathbf{Y}^* = a^2\ p^2 / (p^2+\omega^2) \ . \qquad (10)$$

With the expression (10) we define two numbers. The maximum value A of $\mathbf{Y}.\mathbf{Y}^*$ is found for $\omega = 0$ and is

$$A := (\mathbf{Y}.\mathbf{Y}^*)_{max} = a^2 . \qquad (11)$$

[note: When using a Fourier transform procedure on a computer one finds for **Y** often a sharp spike at $\omega = 0$. This concerns the average value of y(t). Because of our definition of y we expect $\mathbf{Y}(0) = 0$. Of course a spike has to be omitted before the calculation of A.]
The surface B amounts to

$$B := \int \mathbf{Y}(\omega).\mathbf{Y}^*(\omega)\, d\omega \qquad (12)$$

$$= a^2 \int p^2/ (p^2+\omega^2)\, d\omega \ = a^2.p.\pi \ .$$

Next we define the effective width W of the spectrum by

$$W := B / A = p.\pi . \qquad (13)$$

As we have seen the limit of stability is given by $p = 0$. Therefore we may conclude from (13) that the effective width W will indicate how far the system is away from instability. A large value of W indicates a wide spectrum and a large value of p. Then the market reacts stable on incidents. A small value of W indicates a narrow spectrum and a small value of p. Then the market may be near to instability. So the effective width of the power density spectrum of y(t) may be an indicator for the risk of unstable exchange rates. It is not necessary that a crisis occurs exactly at W=0. Near the instability limit a triviality may give the final kick into the unstable domain.

A variant on this is the following. For foreign capital there is a characteristic time T to stay in a country (T in days). We may define this as follows: T = (the total amount of foreign capital invested in the country) / (the net capital flow each day) . Now the expression W' with

$$W' = W.T \qquad (14)$$

is a dimensionless number. Probably W' is a better indicator for instabilities in exchange rates than W . A third indicator may be defined as follows. Establish W_0 , the value of W at a moment when the export and import are in balance. Then the ratio W / W_0 may be an indicator for instability. Has it a critical value at which a crisis is to be expected? Only an empirical investigation of historical crises may be conclusive on what the best variant is.

A second order model

After the simple model of (1) we investigate a second order model. If in the spectrum **Y** specific frequencies occur, one may decide to use a model of second order. We describe the harmonic oscillator with damping

$$d^2y /dt^2 + k.dy /dt + \omega_0^2.y = u . \qquad (15)$$

The parameters k and ω_0 are constant, real numbers. We investigate again the stability with u=0 and a test function $y=\exp(\lambda.t)$. This yields the characteristic equation

$$\lambda^2 + k.\lambda + \omega_0^2 = 0 . \qquad (16)$$

or

$$\lambda_{12} = (-k \pm \sqrt{(k^2 - 4.\omega_0^2)}) / 2 . \qquad (17)$$

We distinguish three situations:

(I) $k^2 < 4\omega_0^2$; oscillation with growth or damping; only stable for $k > 0$

(II) $k^2 = 4\omega_0^2$; critical growth or damping; exponential; stable for $k > 0$

(III) $k^2 > 4\omega_0^2$; exponential behavior; stable for $k > 0$.

In all cases there is only stability when k>0. So the case k = 0 marks the limit of stability.

We calculate the spectrum, in the same way as the derivation of (7):

$$\mathbf{Y}(\omega) = \mathbf{U}(\omega) / ((\omega_0^2 - \omega^2) - i.\omega.k) \qquad (18)$$

and

$$\mathbf{Y}.\mathbf{Y}^* = \mathbf{U}.\mathbf{U}^* / ((\omega_0^2 - \omega^2)^2 + \omega^2.k^2) . \qquad (19)$$

We note that the power density spectrum may have a sharp peak for small values of k. At the limit of stability k=0 we even observe a singularity. Then **Y**.**Y*** goes to infinity when $\omega = \pm\omega_0$. The maximum value A of the power density spectrum (see (11)) may depend strongly on the value of k. Now we will not calculate the surface B (see (12)). But it is easy to verify that the ratio B/A will tend to 0 as the system approaches its limit of stability. In the same way as for the first order system of (1) the effective width W = B / A of the power density spectrum may serve as an indicator for the stability of the system.

A spectrum with more peaks shows that there are several dominant frequencies present in the problem. Then the sharpness of each peak should be investigated separately. The sharper a peak is, the nearer the corresponding oscillation approaches the limit of stability. But it seems unlikely that a diffuse system like a capital market could yield a very refined structure in the spectrum.

Correlation times

As we see a narrow spectrum corresponds with the danger of instability. Let us return to the simple system of (1). The instability limit is p=0. Then dy/dt =0, which means that the system does not return to equilibrium after a disturbance. Generally the threat of instability corresponds with a large time that the system needs to return to equilibrium. Then it should be expected that the value of y(t) correlates with a displaced y(t+τ) after a long displacement time τ. We will derive a correlation function out of the power density spectrum with the aid of a convolution theorem [ref. Butkov, p. 269]. Suppose that we have obtained some power density spectrum **Y**.**Y*** of the free market capital flow. Then, with the method of (4), we calculate the reverse Fourier transform γ:

$$\gamma(\tau) = (1/2\pi) \int \mathbf{Y}(\omega).\mathbf{Y}^*(\omega) \exp(-i.\omega.\tau)\, d\omega \qquad (20)$$

and we substitute **Y*** according to (2) until

$$\gamma(\tau) = (1/2\pi) \int \mathbf{Y}(\omega) \exp(-i.\omega.\tau)\, d\omega \int y(t) \exp(-i.\omega.t)\, dt . \quad (21)$$

We change the order of integration to

$$\gamma(\tau) = (1/2\pi) \int dt\, y(t) \int \mathbf{Y}(\omega) \exp(-i.\omega.(t+\tau))\, d\omega . \qquad (22)$$

We see with (4) that the integral at the right is equal to $2.\pi.y(t+\tau)$ so that

$$\gamma(\tau) = \int y(t).y(t+\tau)\, dt . \qquad (23)$$

The backward transformed power density spectrum is the correlation integral for self correlation after displacement over a time τ . Formula (23) can be seen as the equivalent of a correlation coefficient:

$$\gamma(\tau) \sim \Sigma_i\, y(t_i).y(t_i+\tau)\ . \qquad (24)$$

In formula (23) the function y(t) is correlated with itself after a displacement over a time τ . When one takes a large value of τ , the correlation will be small. When $\tau = 0$ the correlation will be at its maximum. There will exist a typical correlation time Δt at which γ will reach the 1/e value of its maximum:

$$\gamma(\Delta t) / \gamma(0) = 1/e\ . \qquad (25)$$

The characteristic time Δt is nearly the time that disturbances in y(t) need to extinguish.

Finally we mention a general property of spectra. If a spectrum has a width $\Delta\omega \approx W$, this is related to the correlation time Δt by an uncertainty relation

$$\Delta\omega.\Delta t \approx \pi\ . \qquad (26)$$

This relation is universal for Fourier spectra. It may be understood as follows. Two oscillations, the one with angular frequency ω and the other with $\omega+\Delta\omega$ have the same phase in t=0. After a time t the difference in phase φ is $\Delta\varphi = t.\Delta\omega$. When $\Delta\varphi = \pi$, the two oscillations will have lost their phase correlation. Then the time interval has been $\Delta t = \pi/\Delta\omega$, which is the correlation time. If a function y(t) has a typical correlation time Δt, then its spectrum $\mathbf{Y}(\omega)$ has a width of nearly

$$\Delta\omega \approx \pi / \Delta t\ . \qquad (27)$$

For an illustration of this principle we remember the response on a unit step $y = \exp(-p.t)$ of the system of (1). With (13) we see that $p = W/\pi$ and in (26) $\Delta t \approx \pi/W$ so that $y(\Delta t) = \exp(-p.\Delta t) \approx 1/e$. The correlation time is nearly the time needed to let extinguish the disturbance. Now that we have verified (26) for a spectrum around ω=0 we will see what happens if the peak is displaced over a distance ϖ in the spectrum. Then we calculate the new y(t) with (4) and use $\mathbf{Y}=\mathbf{Y}(\omega-\varpi)$. We obtain $y_{new}(t)=y_{old}(t).\exp(-i.\varpi.t)$. A displacement in the spectrum corresponds with an oscillation in the time-signal. The correlation γ of (23) may become an oscillation within an enveloping curve that decreases. Then the enveloping curve retains its characteristic correlation time and the uncertainty relation (26) remains valid.

Let us observe again this uncertainty relation. A narrow spectrum has automatically a large correlation time. Then a system approaches instability as disturbances take a longer time to extinguish. So very generally a narrow spectrum indicates that instability is near. In the same way a wide spectrum corresponds with a short correlation time. Then disturbances extinguish rapidly and the system is stable.

Generalization

From now we forget everything on the special models we have seen. We will try to formulate an approach to predict currency crises. We measure the fluctuation y(t) in the net capital flow that enters a country each day. Only the flow from the free market is taken into account. We determine the spectrum $\mathbf{Y}(\omega)$ and calculate the power density spectrum $\mathbf{Y}.\mathbf{Y}^*$. We calculate the effective profile width W according to the definitions of (11), (12) and

(13). We repeat this procedure for a number of different situations, as normal, stable situations and unstable situations, preceding historic crises of exchange rates. It is our hypothesis now that the effective profile width W, or a derivative of it, may be a good predicting indicator for instabilities in the exchange rates, especially of countries with a serious shortage of export on the balance of payments. Who accepts the challenge to test this hypothesis with historical time series?

Conclusion

When the trade balance of a country has a shortage of export, the incoming capital flow may compensate therefore. Such a situation leads easily to instabilities in exchange rates. Central banks or the IMF may try to stabilize the situation with buffer stocks, but they will lose control when the free market withdraws its capital supply on a large scale. Then a crisis is inevitable. Therefore an indicator will be useful that measures how unstable the market is and so predicts the risk of a collapse. We have proposed to use the width of the Fourier spectrum of the fluctuations in the free market capital flow as an indicator for the stability of this market. Two simple special models indicate that this approach may lead to successful predictions. But also an argument on correlation times in relation to unstable markets confirms that the width of a spectrum may be a reliable tool to establish the risk of a collapse.

Reference

For an introduction to the mathematics of Fourier transforms we refer to Eugene Butkov, MATHEMATICAL PHYSICS, chapter 7, Addison-Wesley Publishing Company, New York 1973 (library of congress catalog card number 68-11391). First edition 1968.